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**NASA  
Technical  
Memorandum**

**NASA TM-82543**



**TIME-DEPENDENT RESPONSE OF FILAMENTARY COMPOSITE  
SPHERICAL PRESSURE VESSELS**

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FILAMENTARY COMPOSITE SPHERICAL PRESSURE  
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16. ABSTRACT <p>A filamentary composite spherical pressure vessel is modeled as a pseudo-isotropic (or transversely isotropic) composite shell, with the effects of the liner and fill tubes omitted. Equations of elasticity, macromechanical and micro-mechanical formulations, and laminate properties are derived for the application of an internally pressured spherical composite vessel. Viscoelastic properties for the composite matrix are used to characterize time-dependent behavior. Using the maximum strain theory of failure, burst pressure and critical strain equations are formulated, solved in the Laplace domain with an associated elastic solution, and inverted back into the time domain using the method of collocation. Viscoelastic properties of HBFR-55 resin are experimentally determined and a Kevlar/HBFR-55 system is evaluated with a FORTRAN program. The computed reduction in burst pressure with respect to time indicates that the analysis employed may be used to predict the time-dependent response of a filamentary composite spherical pressure vessel.</p> <p style="text-align: center;"><b>ORIGINAL PAGE IS OF POOR QUALITY</b></p>					
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## TABLE OF CONTENTS

	Page
LIST OF TABLES.....	vii
LIST OF FIGURES.....	viii
LIST OF SYMBOLS.....	ix
Chapter	
1. INTRODUCTION.....	1
2. MATERIAL MODEL.....	7
3. EQUATIONS OF ELASTICITY.....	8
3.1 Equilibrium Equations.....	8
3.2 Kinematic Equations.....	9
4. MACROMECHANICAL BEHAVIOR OF A LAMINA.....	12
5. MICROMECHANICAL BEHAVIOR OF A LAMINA.....	14
5.1 Determination of $E_L$ .....	16
5.2 Determination of $E_T$ .....	18
5.3 Determination of $\nu_{LT}$ and $\nu_{TL}$ .....	19
5.4 Determination of $\nu_{TT}$ and $G_{LT}$ .....	20
6. PROPERTIES OF THE LAMINATE.....	22
7. MAXIMUM STRAIN THEORY OF FAILURE.....	26
7.1 Boundary Conditions.....	27
7.2 Failure Prediction.....	27
8. DETERMINATION OF VISCOELASTIC MATERIAL PROPERTIES.....	30
8.1 Laplace Transformations.....	30
8.2 Determination of Creep Compliance, $J_m(t)$ .....	34
8.3 Determination of Relaxation Modu- lus, $E_m(t)$ .....	35
8.4 Determination of Poisson's Ratio, $\nu_m(t)$ .....	37



	Page
9. ASSOCIATED ELASTIC SOLUTION.....	38
10. APPLICATION.....	43
10.1 Viscoelastic Material Properties of HBRF-55 Resin.....	43
10.2 Associated Elastic Solution.....	52
10.2.1 Properties of the Laminate.....	52
10.2.2 Maximum Strain Theory of Failure..	54
11. CONCLUSIONS.....	64
BIBLIOGRAPHY.....	66
APPENDIX A. FORTRAN PROGRAM CREEP .....	69
APPENDIX B. FORTRAN PROGRAM THESIS.....	77
APPENDIX C. FORTRAN PROGRAM THESIS.CCC.....	117

## LIST OF TABLES

	Page
TABLE 1. MATERIAL PROPERTIES	44
TABLE 2. EXPERIMENTAL CREEP TEST	47
TABLE 3. VALUES OF $s$ , $E_m(s)$ , and $\nu_m(s)$	55
TABLE 4. VALUES FOR LAMINA PROPERTIES	56
TABLE 5. VALUES FOR LAMINATE PROPERTIES AND $n$	57

# LIST OF FIGURES

	Page
FIGURE 1. CUTAWAY OF TYPICAL FILAMENTARY COMPOSITE SPHERICAL PRESSURE VESSEL	2
FIGURE 2. SCHEMATIC OF MODEL AND DEFINITION OF COORDINATE SYSTEM	3
FIGURE 3. TRANSFORMATION ANGLE BETWEEN L-T AND 1-2 COORDINATE SYSTEMS	23
FIGURE 4. EXPERIMENTAL CREEP TEST APPARATUS	45
FIGURE 5. CREEP COMPLIANCE VS. TIME	48
FIGURE 6. CREEP COMPLIANCE VS. LOG TIME	49
FIGURE 7. RELAXATION MODULUS VS. LOG TIME	51
FIGURE 8. POISSON'S RATIO VS. LOG TIME	53
FIGURE 9. BURST PRESSURE VS. LOG TIME, VARYING FIBER VOLUME FRACTION, $v_f$	59
FIGURE 10. BURST PRESSURE VS. LOG TIME, VARYING $\beta (= r_o/r_i)$	60
FIGURE 11. STRAIN VS. LOG TIME, VARYING FIBER VOLUME FRACTION, $v_f$	61
FIGURE 12. STRAIN VS. LOG TIME, VARYING $\beta (=r_o/r_i)$	62

$a$	Constant [ $= \frac{g}{f} r_o^{-2n} b$ ]
$A$	Constant [ $= p_f(t_o) - \sum_{v=1}^k h_v$ ]
$A_f$	Area of fiber
$A_m$	Area of matrix
$b$	Constant [ $= \frac{p}{gr_i^{(n-\frac{3}{2})} (r_i^{-2n} - r_o^{-2n})}$ ]
$B_j$	Variable in partial fractions inversion [ $= \frac{P(\lambda_j)}{Q'(\lambda_j)}$ ]
$C_i$	Constants in partial fractions inversion
$C_{ijkl}$	Elasticity matrix
$\left. \begin{matrix} C_{\theta\theta}, C_{\theta r}, \\ C_{\theta\phi}, C_{rr} \end{matrix} \right\}$	Stiffness of laminate
$\left. \begin{matrix} C_{LL}, C_{Tr}, \\ C_{RR}, C_{TT}, \\ C_{LT} \end{matrix} \right\}$	Stiffness of lamina
$D$	Constant from least squares curve fit
$E$	Elastic modulus
$E_f$	Elastic modulus of fiber
$E_L$	Elastic modulus of lamina, longitudinal direction
$E_m$	Elastic modulus of matrix

$E_r$	Elastic modulus of laminate, radial direction
$E_T$	Elastic modulus of lamina, transverse direction
$E_\theta$	Elastic modulus of laminate, theta direction
$f$	Constant [ $= (n - \frac{1}{2}) C_{rr} + 2C_{\theta r}$ ]
$F_f$	Fracture strength of fiber
$F_i$	Constant from least squares curve fit
$g$	Constant [ $= (n + \frac{1}{2}) C_{rr} - 2C_{\theta r}$ ]
$G$	Shear modulus
$G_f$	Shear modulus of fiber
$G_m$	Shear modulus of matrix
$G_{LT}, G_{Tr}, G_{Lr}$ }	Shear moduli of lamina
$h_v$	Constant in associated elastic solution
$H$	Constant from least squares curve fit
$K$	Bulk modulus
$K_\theta$	Circumferential strain of composite fiber
$L$	Length of specimen
$L, T, r$	Lamina coordinates
$M$	Constant [ $= 3, 4, 5, \dots$ ]
$n$	Degree of anisotropy
$N$	Constant [ $= \frac{G_m(\pi+4v_f) + G_m(\pi-4v_f)}{G_f(\pi-4v_f) + G_m(\pi+4v_f)}$ ]
$P$	Force
$p$	Pressure

$P_f$	Burst pressure
$P(\lambda_j)$	Polynomial in partial fractions inversion
$Q(\lambda_j)$	Polynomial in partial fractions inversion
$R_i$	Assigned values in least squares curve fit
$r_i$	Inner radius of composite
$r_o$	Outer radius of composite
$r, \theta, \phi$	Spherical coordinates of laminate
$s$	Laplace parameter
$S_{ij}$	Compliance matrix
$t$	Time
$t_o$	Time zero
$u$	Displacement
$u_r$	Displacement in radial direction
$u_\theta$	Displacement in theta direction
$u_\phi$	Displacement in phi direction
$v_f$	Volume fraction of fiber
$v_m$	Volume fraction of matrix
$W$	Width of specimen
$W_f$	Width of fiber
$W_m$	Width of matrix
$\alpha$	Angle between L axis and l axis, or between adjacent lamina
$\alpha_v$	Constant [ $= e^{(7-2v)}$ ]

$\beta$	Ratio of outer radius to inner radius ( $r_o/r_i$ )
$\gamma$	Wrap angle
$\epsilon_f$	Strain in fiber
$\epsilon_{ij}$	Strain components
$\epsilon_L$	Strain in lamina, longitudinal direction
$\epsilon_m$	Strain in matrix
$\epsilon_o$	Constant uniaxial strain
$\epsilon_T$	Strain in lamina, transverse direction
$\left. \begin{array}{l} \epsilon_{\theta\theta}, \epsilon_{\phi\phi}, \\ \epsilon_{\phi r}, \epsilon_{r\theta}, \\ \epsilon_{rr}, \epsilon_{\theta\phi} \end{array} \right\}$	Strain components in laminate
$\lambda_j$	Roots of partial fractions solution
$\Lambda$	Constant [ $= (1 + \nu_{TT})(1 - \nu_{TT} - 2\nu_{LT} - \nu_{TL})^{-1}$ ]
$\nu$	Poisson's ratio
$\nu_f$	Poisson's ratio of fiber
$\nu_m$	Poisson's ratio of matrix
$\left. \begin{array}{l} \nu_{LT}, \nu_{TT}, \\ \nu_{TL} \end{array} \right\}$	Poisson's ratio of lamina
$\sigma$	Average stress
$\sigma_f$	Stress in fiber
$\sigma_L$	Stress in lamina, longitudinal direction
$\sigma_m$	Stress in matrix

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$\sigma_T$	Stress in lamina, transverse direction
$\sigma_{ij}$	Stress components
$\sigma_o$	Constant uniaxial stress
$\left. \begin{array}{l} \sigma_{\phi r}, \sigma_{r\theta}, \\ \sigma_{\phi\phi}, \sigma_{\theta\phi}, \\ \sigma_{rr}, \sigma_{\theta\theta} \end{array} \right\}$	Stress components in laminate
$\pi$	Constant [ = 3.14... ]
$-$	Denotes Laplace transform when placed over a symbol

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## CHAPTER 1. INTRODUCTION

Pressure vessels which are made of filamentary composites have had a profound impact in applications where weight efficiency is a factor. In particular, filamentary composite pressure vessels with a spherical geometry are used in applications of space shuttle tankage [1] and for storage of various fluids [2].

Spherical storage pressure vessels usually consist of a thin metallic bladder onto which is wound high-strength fibers bound by an organic matrix. The bladder, which acts as a liner to prevent leakage, is usually fitted with one or more fill tubes for introducing the fluid to be pressurized. As shown in Figure 1, the spherical shape allows filament winding patterns which result in a laminate which may be considered as pseudo-isotropic. That is, the pseudo-isotropic (transversely isotropic) condition is met if the angles between lamina in a laminate satisfy the relation [3]:

$$\gamma = \frac{\pi}{M} \quad (1)$$

where  $M = 3, 4, 5, \dots$

This wrap angle ensures that the stiffness matrix will be invariant for the  $\theta$ - $\phi$  plane shown in Figure 2 and the composite may be modeled as a transversely isotropic material.

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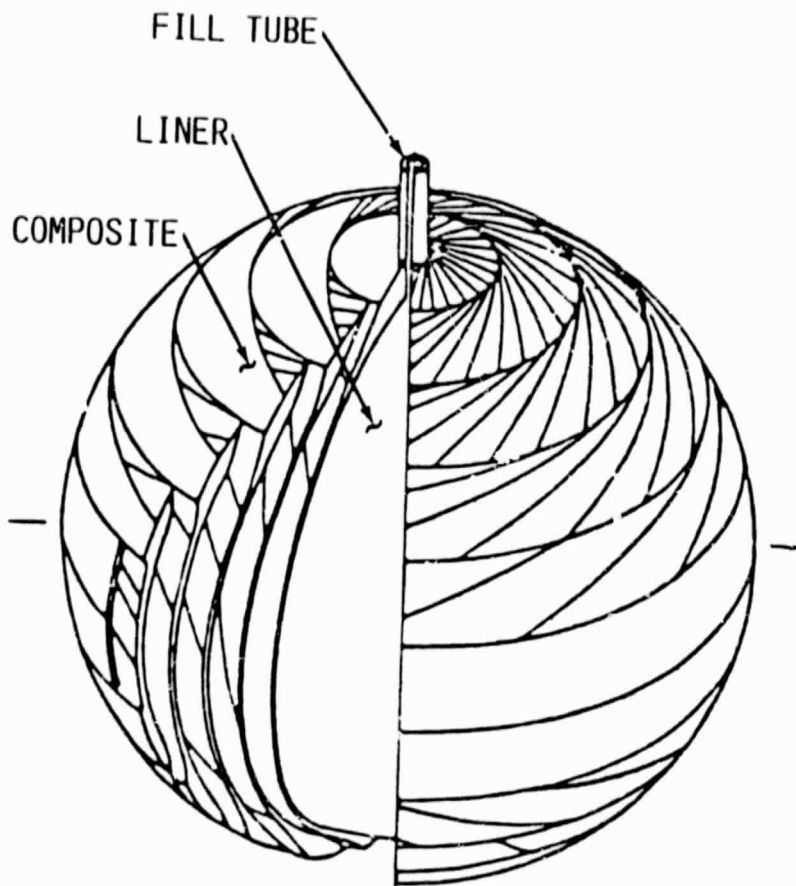


FIGURE 1. Cutaway of typical filamentary composite spherical pressure vessel [7]

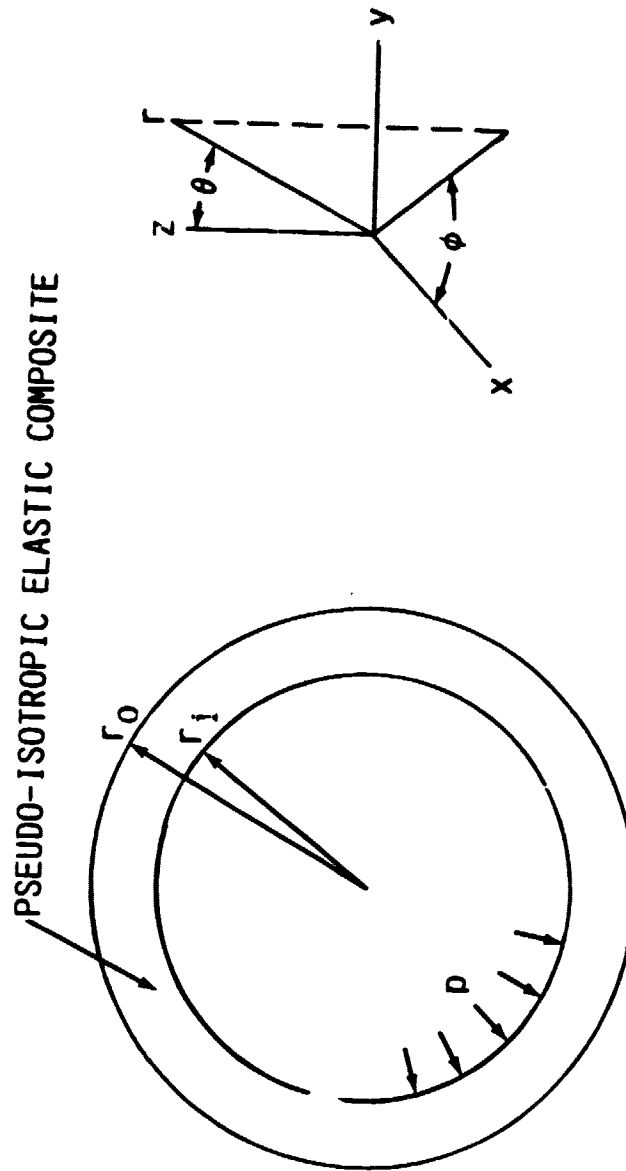


FIGURE 2. Schematic of model and definition of coordinate system

A number of closed-form solutions have been developed [3,4] which predict the stress-strain behavior of a composite shell assuming a pseudo-isotropic material and an elastic-plastic bladder response. These solutions predict short-term failure pressures of the vessel, and correlate well with experimental data [4,5,6]. Finite-element techniques are also used [2,7] to model discontinuities near the fill tubes.

Although recognized as a significant problem, little is known about the mechanisms which lead to the loss of long-term structural integrity of filamentary composite spherical pressure vessels. Some possible causes of this time-dependent loss of structural integrity are environmental degradation, matrix crazing, and manufacturing processes.

The static fatigue susceptibility in composites is well-known, but it has not been treated successfully from the standpoint of mechanism identification. Therefore, in the case of long-term, high-pressurization storage, high factors of safety are used to compensate for uncertainties regarding creep rupture under conditions of sustained pressurization. The resulting low design allowables then detract from the weight-saving attributes of composites. This problem has heretofore been dealt with by a statistical analysis of manufacturing variations and experimental data [8].

The study of the time-dependent behavior of filamentary composite pressure vessels is further complicated by the lack of experimental data. Some data were collected at Lawrence Livermore National Laboratory on S-glass/epoxy composites [9], but most of the test specimens were destroyed and the test data invalidated due to a major earthquake in Northern California in 1980.

This study examines the time-dependent response of filamentary composite spherical pressure vessels by considering the linearly viscoelastic character of the matrix in the filamentary composite laminate. This characterization is important because the composite material over a period of time becomes more anisotropic, thus increasing the failure potential of the system. Therefore, this material model which includes the time-dependent properties of the matrix in a composite laminate is used to predict burst pressure and strain under conditions of sustained internal pressurization of a spherical pressure vessel. The lamina and laminate properties in the analysis are initially determined using micromechanical and macro-mechanical techniques. Viscoelastic material properties for the matrix are experimentally determined and included in the constitutive equations for the composite. These time-dependent expressions are transformed into the Laplace domain and solved using the maximum strain theory of failure. The solution is then inverted back into the time domain using the method of collocation, and burst pressure and critical strain

of the composite spherical pressure vessels are expressed as a function of time. Finally, an application is given for a Kevlar/epoxy system, where the solution is numerically evaluated. The burst pressures for this system are predicted as a function of time, and critical strain is computed for a constant internal pressurization. The effects of constituent volume fractions and shell thickness on failure potential are also investigated. Thus, an analytical method is outlined and a numerical application is given to predict the time-dependent response of a filamentary composite spherical pressure vessel.

## CHAPTER 2. MATERIAL MODEL

Filamentary composite spherical pressure vessels generally have filaments which are wound into a laminate considered as pseudo-isotropic (or transversely isotropic). In this analysis, the filamentary composite pressure vessel is assumed to be transversely isotropic in the  $\theta - \phi$  plane, as shown in Figure 2.

The spherical geometry of pressure vessels under consideration is uniform, except in the areas surrounding the fill tubes. However, the model used in this analysis assumes a uniformly spherical geometry and does not include the discontinuities in the fill tube regions.

It has been shown that the bladder which lines the inner surface of the composite shell has a minimal effect in the burst pressure prediction of the pressure vessel [4, 10]. Therefore, the structural model used in this analysis does not include the effect of the bladder. Models have been developed which consider the elastic-plastic behavior of the bladder in failure prediction [3].

This model will be used in the following viscoelastic analysis to predict burst pressure and to calculate critical strain. It is further elaborated upon in the application to a Kevlar/epoxy spherical pressure vessel.

### CHAPTER 3. EQUATIONS OF ELASTICITY

For a spherical geometry with uniformly symmetrical loading, all of the shear stress and strain components vanish. For the special case of a transversely isotropic material system, the general governing equations of elasticity may be further simplified. The following discussion derives the equilibrium equations and kinematic equations in spherical coordinates for a sphere transversely isotropic in the  $r$  plane.

#### 3.1 EQUILIBRIUM EQUATIONS

Neglecting body forces, the following general equilibrium equations relative to a spherical coordinate system are [11]:

$$\frac{\partial \sigma_{\phi}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{1}{r \sin \phi} \frac{\partial \sigma_{\theta\phi}}{\partial \theta} + \frac{3}{r} \sigma_{r\phi} + \frac{\cos \phi}{r \sin \phi} (\sigma_{\phi\phi} - \sigma_{\theta\theta}) = 0 \quad (2a)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\phi}}{\partial \phi} + \frac{1}{r \sin \phi} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{3}{r} \sigma_{r\theta} + \frac{2 \cos \phi}{r \sin \phi} \sigma_{\theta\phi} = 0 \quad (2b)$$

$$\begin{aligned} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\phi r}}{\partial \phi} + \frac{1}{r \sin \phi} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\cos \phi}{r \sin \phi} \sigma_{r\phi} + \\ \frac{1}{r} [2\sigma_{rr} - \sigma_{\phi\phi} - \sigma_{\theta\theta}] = 0 \end{aligned} \quad (2c)$$

where  $\sigma_{ij}$  are stress components in the  $\theta, \phi, r$  coordinate system. For uniform loading conditions, all of the shear stress components vanish, i.e.,  $\sigma_{\phi r} = \sigma_{r\phi} = \sigma_{r\theta} = \sigma_{\theta r} = \sigma_{\phi\theta} = \sigma_{\theta\phi} = 0$ . Therefore, Equations (2) reduce to:



$$\frac{1}{r} \frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{\cos \phi}{r \sin \phi} (\sigma_{\phi\phi} - \sigma_{\theta\theta}) = 0 \quad \text{ORIGINAL PAGE IS OF POOR QUALITY} \quad (3a)$$

$$\frac{1}{r \sin \phi} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \quad (3b)$$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (2\sigma_{rr} - \sigma_{\phi\phi} - \sigma_{\theta\theta}) = 0 \quad (3c)$$

For a transversely isotropic material system, the tangential stresses,  $\sigma_{\theta\theta}$  and  $\sigma_{\phi\phi}$  are equal. Equations (3) thus can be written:

$$\frac{1}{r} \frac{d\sigma_{\phi\phi}}{d\phi} = 0 \quad (4a)$$

$$\frac{1}{r \sin \theta} \frac{d\sigma_{\theta\theta}}{d\theta} = 0 \quad (4b)$$

$$\frac{d\sigma_{rr}}{dr} + \frac{2}{r} (\sigma_{rr} - \sigma_{\theta\theta}) = 0 \quad (4c)$$

Equation (4c) is then the only equilibrium equation of interest.

### 3.2 KINEMATIC EQUATIONS

For linear elasticity, the kinematic equations in spherical coordinates for any geometry are given by [11]:

$$\epsilon_{\phi\phi} = \frac{1}{r} \frac{\partial u_{\phi}}{\partial \phi} + \frac{u_r}{r} \quad (5a)$$

$$\epsilon_{\theta\theta} = \frac{1}{r \sin \phi} \frac{\partial u_{\theta}}{\partial \phi} + \frac{u_r}{r} + \frac{u_{\phi}}{r} \cot \phi \quad (5b)$$

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r} \quad (5c)$$

$$\epsilon_{\theta\phi} = \frac{1}{2} \left( \frac{1}{r \sin \phi} \frac{\partial u_{\phi}}{\partial \theta} - \frac{u_{\theta}}{r} \cot \phi + \frac{1}{r} \frac{\partial u_{\theta}}{\partial \phi} \right) \quad (5d)$$

$$\epsilon_{\phi r} = \frac{1}{2} \left( \frac{1}{r} \frac{\partial u_r}{\partial \phi} - \frac{u_{\phi}}{r} + \frac{\partial u_{\phi}}{\partial r} \right) \quad (5e)$$

$$\epsilon_{r\theta} = \frac{1}{2} \left( \frac{\partial u_{\theta}}{\partial r} + \frac{1}{r \sin \phi} \frac{\partial u_r}{\partial \theta} - \frac{u_{\theta}}{r} \right) \quad (5f)$$

where  $\epsilon_{ij}$  are the strain components and  $u_i$  are the displacements in the  $\theta, \phi, r$  coordinate system.

For a condition of uniform loading, all shear strain components vanish and Equations (5d) through (5f) may then be eliminated. Also, displacement components  $u_{\phi}$  and  $u_{\theta}$  are zero and Equations (5a) through (5c) reduce to:

$$\epsilon_{\phi\phi} = \frac{u_r}{r} \quad (6a)$$

$$\epsilon_{\theta\theta} = \frac{u_r}{r} \quad (6b)$$

$$\epsilon_{rr} = \frac{du_r}{dr} \quad (6c)$$

For a material transversely isotropic in the  $r$  plane,

$\epsilon_{\theta\theta} = \epsilon_{\phi\phi}$  and  $u_r = u$ . Therefore, the final form of the kinematic equations is:

$$\epsilon_{\theta\theta} = \frac{u}{r} \quad (7a)$$

$$\epsilon_{rr} = \frac{du}{dr} \quad (7b)$$

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## CHAPTER 4. MACROMECHANICAL BEHAVIOR OF A LAMINA

A lamina in a filamentary composite is an assemblage of fibers embedded in a matrix. It is necessary to describe the macromechanical behavior of a lamina, i.e., the behavior when averaged properties are considered, in order to understand the laminated structure. In this discussion the coordinate system  $\theta, \phi, r$  is aligned with the principal material directions 1,2,3 of the lamina.

The generalized Hooke's law relating stresses to strains can be written as:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad i, j, k, l = 1, 2, \dots, 6 \quad (8)$$

where  $\sigma_{ij}$  are stress components,  $C_{ijkl}$  is the elasticity matrix, and  $\epsilon_{kl}$  are the strain components. Although the elasticity matrix has 36 terms, it is symmetric and only 21 of the terms are independent.

The 21 constants in the elasticity matrix are necessary in order to characterize a general anisotropic material because there are no planes of symmetry. The case of a lamina which is transversely isotropic in the 3 plane yields properties which are non-directional in the 1-2 plane. The stress-strain relationship is based upon only five independent constants and is given by [12]:

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$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{23} \\ \epsilon_{31} \\ \epsilon_{12} \end{bmatrix} \quad (9)$$

Letting subscript 1 =  $\theta$ , subscript 2 =  $\phi$ , and subscript 3 =  $r$  in spherical coordinates, the stress-strain relations become:

$$\begin{bmatrix} \sigma_{\theta\theta} \\ \sigma_{\phi\phi} \\ \sigma_{rr} \end{bmatrix} = \begin{bmatrix} C_{\theta\theta} & C_{\theta\phi} & C_{\theta r} \\ C_{\theta\phi} & C_{\theta\theta} & C_{\theta r} \\ C_{\theta r} & C_{\theta r} & C_{rr} \end{bmatrix} \begin{bmatrix} \epsilon_{\theta\theta} \\ \epsilon_{\phi\phi} \\ \epsilon_{rr} \end{bmatrix} \quad (10)$$

Again, due to loading symmetry,  $\epsilon_{\theta\theta} = \epsilon_{\phi\phi}$ . Therefore, the constitutive equations reduce to:

$$\sigma_{\theta\theta} = (C_{\theta\theta} + C_{\theta\phi}) \epsilon_{\theta\theta} + C_{\theta r} \epsilon_{rr} \quad (11a)$$

and

$$\sigma_{rr} = 2 C_{\theta r} \epsilon_{\theta\theta} + C_{rr} \epsilon_{rr} \quad (11b)$$

## CHAPTER 5. MICROMECHANICAL BEHAVIOR OF A LAMINA

Micromechanics is the study of composite behavior where the interaction of the constituent materials is examined in detail. This study is based on a lamina consisting of two constituents, the fiber and the matrix. In the coordinate system,  $L, T, r$ , the longitudinal direction,  $L$ , is along the length of the fiber, and the transverse direction,  $T$ , is across the width of the fiber and is orthogonal to  $L$ . The radial direction,  $r$ , is perpendicular to the  $L$ - $T$  plane.

The constitutive relationship for an individual lamina in the  $\theta$ - $\phi$  plane may be expressed by the matrix given in Equation (10). If the coordinate axes ( $L, T, r$ ) are oriented in the principal material directions and assumed to be transversely isotropic in the  $r$ -plane perpendicular to the fiber direction, the elasticity matrix is [3]:

$$[C] = \begin{bmatrix} C_{LL} & C_{LT} & C_{Lr} & 0 & 0 & 0 \\ C_{LT} & C_{TT} & C_{Tr} & 0 & 0 & 0 \\ C_{Lr} & C_{Tr} & C_{rr} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{Tr} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{LT} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{Lr} \end{bmatrix} \quad (12)$$

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Because of isotropy in the T-r plane:

$$C_{Lr} = C_{LT} \quad (13a)$$

$$C_{rr} = C_{TT} \quad (13b)$$

$$G_{Lr} = G_{LT} \quad (13c)$$

and

$$G_{Tr} = \frac{C_{TT} - C_{Tr}}{2} \quad (13d)$$

The corresponding lamina compliance matrix is:

$$[S] = \begin{bmatrix} \frac{1}{E_L} - \frac{\nu_{TL}}{E_T} - \frac{\nu_{TL}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} - \frac{\nu_{TT}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} - \frac{\nu_{TT}}{E_T} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{Tr}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} \quad (14a)$$

Inversion of the lamina compliance matrix yields the elasticity matrix, Equation (12), and the following relationships [13]:

$$\begin{aligned}
C_{LL} &= (1 - \nu_{TT}^2) \Lambda E_L \\
C_{TT} &= (1 - \nu_{LT} \nu_{TL}) \Lambda E_T \\
C_{LT} &= \nu_{TL} (1 + \nu_{TT}) \Lambda E_L \\
C_{Tr} &= (\nu_{TT} + \nu_{LT} \nu_{TL}) \Lambda E_T \\
G_{Tr} &= \frac{(1 - \nu_{TT} - 2\nu_{LT} \nu_{TL}) \Lambda E_T}{2} \quad (14b)
\end{aligned}$$

where

$$\Lambda = [(1 + \nu_{TT})(1 - \nu_{TT} - 2\nu_{LT} \nu_{TL})]^{-1}$$

The lamina engineering constants ( $E_L$ ,  $E_T$ ,  $\nu_{LT}$ ,  $\nu_{TL}$ ,  $\nu_{TT}$ ,  $G_{LT}$ ) can be expressed in terms of the elastic properties ( $E$ ,  $\nu$ ,  $G$ ) and volume fractions of the fiber and matrix of the composite material. Micromechanical techniques are used which are based upon the assumption that the properties of the fiber and matrix are isotropic and are equal in tension and compression.

### 5.1 DETERMINATION OF $E_L$

For a volume of material loaded in the longitudinal direction, basic mechanics of materials says that

$$\epsilon_L = \frac{\Delta L}{L} \quad (15)$$

where  $L$  is the specimen length and  $\epsilon_L$  is the longitudinal extensional strain.



The stresses in the fiber and the matrix are given by

$$\sigma_f = E_f \epsilon_L \quad (16a)$$

$$\epsilon_m = E_m \epsilon_L \quad (16b)$$

where  $E_f$  is the fiber modulus and  $E_m$  is the matrix modulus. The average stress  $\sigma$  acts on a cross-sectional area  $A$ ,  $\sigma_f$  acts on the cross-sectional area of the fiber,  $A_f$ , and  $\sigma_m$  acts on the cross-sectional area of the matrix,  $A_m$ . Therefore, the resultant force on the composite material is:

$$P = \sigma A = \sigma_f A_f + \sigma_m A_m \quad (17)$$

knowing that

$$\sigma = E_L \epsilon_L, \quad (18)$$

substituting Equations (16) and (18) into Equation (17) and rearranging yields:

$$E_L = E_f \frac{A_f}{A} + E_m \frac{A_m}{A} \quad (19)$$

Defining the volume fractions of the fiber and matrix, respectively,

$$v_f = \frac{A_f}{A} \text{ and } v_m = \frac{A_m}{A} \quad (20)$$

and substituting Equations (20) into Equation (19) results in the expression:

$$E_L = E_f v_f + E_m v_m \quad (21)$$

Equation (21) is the "rule of mixtures" expression for the elastic modulus in the direction of the fibers.

## 5.2 DETERMINATION OF $E_T$

If a stress  $\sigma_T$  is applied in a direction transverse to the fibers, the strains in the fiber and in the matrix are given by:

$$\epsilon_f = \frac{\sigma_T}{E_f} \quad (22a)$$

$$\epsilon_m = \frac{\sigma_T}{E_m} \quad (22b)$$

Let  $W$  be the transverse dimension of the specimen. Therefore, on an average basis,  $\epsilon_f$  acts on  $v_f W$  and  $\epsilon_m$  acts on  $v_m W$ . Thus, the total transverse deformation is

$$\epsilon_T W = v_f W \epsilon_f + v_m W \epsilon_m \quad (23)$$

or

$$\epsilon_T = v_f \epsilon_f + v_m \epsilon_m \quad (24)$$

where  $\epsilon_T$  is the transverse extensional strain.

Substituting Equations (22) into Equation (24) gives

$$\epsilon_T = v_f \frac{\sigma_T}{E_f} + v_m \frac{\sigma_T}{E_m} \quad (25)$$

$$\text{Knowing that } \sigma_T = E_T \epsilon_T, \quad (26)$$

and substituting this relationship into Equation (25) yields

$$\sigma_T = E_T \left( v_f \frac{\sigma_T}{E_f} + v_m \frac{\sigma_T}{E_m} \right) \quad (27)$$

Solving for  $E_T$  from Equation (27) provides the expression for the transverse modulus:

$$E_T = \frac{E_f E_m}{v_m E_f + v_f E_m} \quad (28)$$

### 5.3 DETERMINATION OF $\nu_{LT}$ AND $\nu_{TL}$

The major Poisson's ratio,  $\nu_{LT}$ , can be determined by a method similar to that used to determine  $E_L$ . For a volume of material loaded in the longitudinal direction, the Poisson's ratio is defined as:

$$\nu_{LT} = - \frac{\epsilon_T}{\epsilon_L} \quad (29)$$

For a deformation in the transverse direction:

$$\Delta W = - W \epsilon_T \quad (30a)$$

or

$$\Delta W = W \nu_{LT} \epsilon_L \quad (30b)$$

The total deformation is also given by

$$\Delta W = \Delta W_f + \Delta W_m \quad (31)$$

where  $W_f$  is the fiber width and  $W_m$  is the matrix width.

On the average basis,  $\epsilon_f$  acts on  $v_f W$  and  $\epsilon_m$  acts on  $v_m W$ .

Therefore,

$$\Delta W_f = W v_f \epsilon_f = W v_f v_f \epsilon_L \quad (32a)$$

and

$$\Delta W_m = W v_m \epsilon_m = W v_m v_m \epsilon_L \quad (32b)$$

where  $v_f$  is the Poisson's ratio of the fiber and  $v_m$  is the Poisson's ratio of the matrix.

Then,

$$\Delta W = W (v_m v_m \epsilon_L + v_f v_f \epsilon_L) \quad (33)$$

Substituting Equation (30) into Equation (33) gives

$$v_{LT} \epsilon_L = \epsilon_L (v_m v_m + v_f v_f) \quad (34)$$

which reduces to the expression:

$$v_{LT} = v_m v_m + v_f v_f \quad (35)$$

Equation (35) is the "rule of mixtures" expression for the major Poisson's ratio.

Because of symmetry requirements of the lamina compliance matrix,  $v_{TL}$  can be obtained from the relationship:

$$v_{TL} = \frac{E_T}{E_L} v_{LT} \quad (36)$$

#### 5.4 DETERMINATION OF $v_{TT}$ AND $G_{LT}$

Although there is not general agreement about the prediction of transverse lamina properties, reasonable predictions may be expressed as [14,15]:

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$$v_{TT} = v_f v_f + v_m v_m \frac{1 + v_m - v_{LT} \left( \frac{E_m}{E_L} \right)}{1 - v_m^2 + v_m v_{LT} \left( \frac{E_m}{E_L} \right)} \quad (37)$$

$$G_{LT} = \frac{G_m}{2} \frac{(4-\pi) + \pi N}{4} + \frac{4N}{(4-\pi) N + \pi} \quad (38)$$

where

$$N = \frac{G_f(\pi+4v_f) + G_m(\pi-4v_f)}{G_f(\pi-4v_f) + G_m(\pi+4v_f)} \quad (39)$$

Laminate properties may be determined by averaging properties of the individual lamina [12]. The laminate stiffnesses ( $C_{rr}$ ,  $C_{r\theta}$ ,  $C_{\theta\theta}$ ,  $C_{\theta\phi}$ ) are obtained by approximating the various lamina orientations in the spherical geometry as a continuous function of  $\alpha$ , shown in Figure 3. An average stiffness is then computed by integrating the lamina stiffnesses which have been transformed from the (L,T,r) coordinates to the (1,2,r) coordinate system, also shown in Figure 3. These averaged elastic properties are derived with the assumption of constant strain in the laminate, independent of the orientation.

The average laminate stiffness in the ( $\theta, \phi, r$ ) coordinate system is calculated from:

$$C_{\theta\theta} = \frac{1}{\pi} \int_0^{\pi} C_{11} d\alpha \quad (40a)$$

$$C_{\theta\phi} = \frac{1}{\pi} \int_0^{\pi} C_{12} d\alpha \quad (40b)$$

$$C_{\theta r} = \frac{1}{\pi} \int_0^{\pi} C_{1r} d\alpha \quad (40c)$$

$$\text{and } C_{rr} = C_{TT} \quad (40d)$$

These equations may be used to determine the laminate elastic stiffness for any thickness, if the individual lamina is thin compared to its width.

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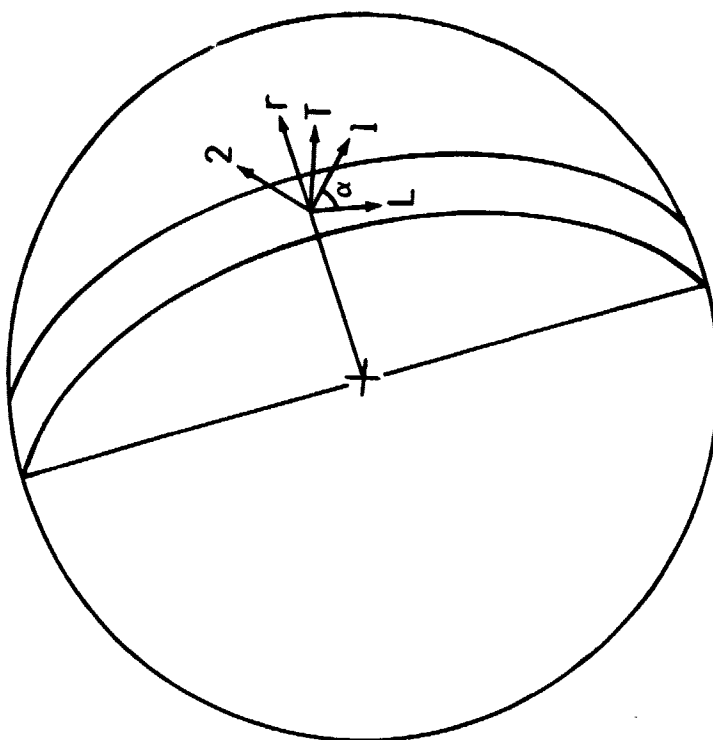


FIGURE 3. Transformation angle between L-T and 1-2 coordinate systems

Using the method outlined by Gerstle [3], the following equations are obtained:

$$C_{\theta\theta} = \frac{3C_{LL}}{8} + \frac{3C_{TT}}{8} + \frac{1}{4} (C_{LT} + 2G_{LT}) \quad (41a)$$

$$C_{\theta\phi} = \frac{C_{LL}}{8} + \frac{C_{TT}}{8} + \frac{1}{4} (3C_{LT} - 2G_{LT}) \quad (41b)$$

$$C_{\theta r} = \frac{1}{2} (C_{LT} + C_{Tr}) \quad (41c)$$

Assuming that the transversely isotropic condition is met in the  $\theta - \phi$  plane

$$C_{\phi\phi} = C_{\theta\theta} \quad (42a)$$

$$G_{\theta\phi} = \frac{C_{\theta\theta} - C_{\theta\phi}}{2} \quad (42b)$$

$$C_{\phi r} = C_{\theta r} \quad (42c)$$

By substituting Equations (14) into Equations (41), the laminate stiffnesses may be obtained in terms of the elastic constants of an individual lamina:

$$C_{\theta\theta} = \frac{\Lambda}{8} [3(1-\nu_{TT}^2)E_L + 3(1-\nu_{LT}\nu_{TL})E_T + 2\nu_{TL}(1+\nu_{TT})E_L] + \frac{G_{LT}}{2} \quad (43a)$$

$$C_{\theta\phi} = \frac{\Lambda}{8} [(1-\nu_{TT}^2)E_L + (1-\nu_{LT}\nu_{TL})E_T + 6\nu_{TL}(1+\nu_{TT})E_L] + \frac{G_{LT}}{2} \quad (43b)$$

$$C_{\theta r} = \frac{E_T \Lambda}{2} [\nu_{LT}(1-\nu_{TT}) + (\nu_{TT} + \nu_{LT}\nu_{TL})] \quad (43c)$$

$$C_{rr} = E_T \Lambda (1-\nu_{LT}\nu_{TL}) \quad (43d)$$



The engineering constraints of the laminate are then obtained from:

$$E_{\theta\theta} = \frac{(C_{\theta\theta} - C_{\theta\phi}) [C_{rr}(C_{\theta\theta} + C_{\theta\phi}) - 2 C_{\theta r}^2]}{C_{\theta\theta} C_{rr} - C_{\theta r}^2} \quad (44)$$

$$E_{rr} = C_{rr} - \frac{2 C_{\theta r}^2}{C_{\theta\theta} + C_{\theta\phi}} \quad (45)$$

$$v_{\theta\phi} = \frac{C_{\theta\phi} C_{rr} - C_{\theta r}^2}{C_{\theta\theta} C_{rr} - C_{\theta r}^2} \quad (46)$$

$$v_{\theta r} = \frac{C_{\theta r} (C_{\theta\theta} - C_{\theta\phi})}{C_{\theta\theta} C_{rr} - C_{\theta r}^2} \quad (47)$$

Finally, the degree of anisotropy of the laminate is obtained from the relationship:

$$n = \frac{3}{2} \left( \frac{E_{\theta\theta}}{E_{rr}} \right)^{\frac{1}{2}} \quad (48)$$

It can be noted that if  $n = 1.5$  the composite material is isotropic. The magnitude of  $n$  is affected by factors such as wrap angle, fiber and matrix content, and, as will be demonstrated herein, the time-dependent properties of the matrix material.

## CHAPTER 7. MAXIMUM STRAIN THEORY OF FAILURE

Prediction of the burst (failure) pressure of a filamentary composite spherical pressure vessel with a metallic bladder is a function of the elastic-plastic behavior of the metallic bladder as well as of the composite material, but it has been demonstrated [4,10] that the behavior of the bladder has relatively little effect on the failure prediction of the composite. Therefore, the effect of the bladder on the burst pressure or circumferential strain of the composite sphere will not be considered in the following formulation.

Prediction of the burst pressure requires a determination of the maximum values of stress and strain at the inner surface of the sphere. The maximum strain theory of failure is chosen for this study for the following reasons:

- 1) Considerable experimental data has shown the general applicability of the maximum strain criterion for predicting vessel failure.

- 2) The published values for composite maximum strain have been shown to be essentially independent of vessel configuration or fiber volume fraction.

- 3) The calculated maximum strain includes the contribution of radial stress. This is important in predicting the behavior of thick-walled vessels.

## 7.1 BOUNDARY CONDITIONS

Boundary conditions for the composite material portion of the pressurized structure are:

$$\sigma_{rr} = -p \text{ at the inner surface, } r = r_i \quad (49a)$$

$$\sigma_{rr} = 0 \text{ at the outer surface, } r = r_o \quad (49b)$$

where  $\sigma_{rr}$  is the radial stress component and  $p$  is the internal pressure.

These boundary conditions and the symmetry of the spherical geometry greatly simplify the analysis and preclude the necessity of a three-dimensional analysis.

## 7.2 FAILURE PREDICTION

Within the composite, Equations (4), (7), and (11) yield:

$$C_{rr} \frac{d^2 u}{dr^2} + \frac{2 C_{rr}}{r} \frac{du}{dr} + 2 (C_{r\theta} - C_{\theta\theta} - C_{\theta\phi}) \frac{u}{r^2} = 0 \quad (50)$$

The solution to this second order differential equation is:

$$u = ar^{n-\frac{1}{2}} + br^{-n-\frac{1}{2}} \quad (51)$$

where  $a$  and  $b$  are constants given by:

$$a = \frac{g}{f} r_o^{-2n} b \quad (52)$$

$$b = \frac{p}{g r_i^{(n-\frac{3}{2})} (r_i^{-2n} - r_o^{-2n})} \quad (53)$$

and  $f$  and  $g$  are constants given by:

$$f = (n - \frac{1}{2}) C_{rr} + 2C_{\theta r} \quad (54)$$

$$g = (n + \frac{1}{2}) C_{rr} - 2C_{\theta r} \quad (55)$$

The maximum strain prediction is found from:

$$a r_i^{(n-\frac{3}{2})} + b r_i^{(-n-\frac{3}{2})} = K_\theta \quad (56)$$

where  $K_\theta$  is the ultimate composite tensile strain. It is reasonable to assume that the maximum strain in the structure will be no greater than the ultimate tensile strain in the fibers. Experimental results from biaxially loaded pseudo-isotropic cylinders [3] have shown failure strains of about  $0.75 \epsilon_f$ , where  $\epsilon_f = \frac{F_f}{E_f}$  if the fibers remain linear to failure. Then,

$$K_\theta \cong 0.75 \frac{F_f}{E_f} \quad (57)$$

where  $E_f$  = Fiber modulus of elasticity

$F_f$  = Fiber fracture strength

The maximum strain theory of failure predicts that vessel failure occurs when the circumferential strain in the composite is equal to or greater than the composite ultimate tensile strain,  $K_\theta$ . This failure criterion may be expressed as:

$$\epsilon_{\theta\theta} \geq K_\theta \quad (58)$$

The composite strain,  $\epsilon_{\theta\theta}$ , is found by evaluating kinematic Equation (5b) for the displacement given in Equation (6b).

Thus,

$$\epsilon_{\theta\theta} = \frac{p}{g(\beta^{2n}-1)} \left( \frac{g}{f} \right) + \beta^{2n} \quad (59)$$

where  $\beta$  is the ratio  $r_o / r_i$ .

Letting the pressure,  $p$ , equal the burst pressure,  $p_f$ , and comparing  $K_\theta$  and  $\epsilon_{\theta\theta}$ , the following equation can be used to determine when composite failure will occur:

$$K_\theta \leq \frac{p_f}{g(\beta^{2n}-1)} \left( \frac{g}{f} + \beta^{2n} \right) \quad (60)$$

If failure occurs when  $K_\theta$  equals  $\epsilon_{\theta\theta}$ , the expression for burst pressure is given by:

$$p_f = K_\theta g \frac{(\beta^{2n}-1)}{\frac{g}{f} + \beta^{2n}} \quad (61)$$

With the maximum strain theory of failure, Equations (60) and (61) are used to predict composite failure for a transversely isotropic spherical pressure vessel. These equations form the basis for the following derivations which consider the time-dependent behavior of the composite.

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## CHAPTER 8. DETERMINATION OF VISCOELASTIC MATERIAL PROPERTIES

A stress analysis of viscoelastic materials requires the use of mechanical properties which characterize the time-dependent behavior of the materials. These material properties must be experimentally determined. In this discussion of the properties of a linearly viscoelastic material, only the epoxy matrix of the composite is considered to exhibit time-dependent behavior.

The determination of the relaxation modulus,  $E_m(t)$ , is fundamental to the characterization of a linearly viscoelastic material. In a standard creep test, a constant uniaxial stress  $\sigma_0$  is applied and the time-dependent uniaxial strain is measured. The time-dependent strain is proportional to  $\sigma_0$  and may be written as

$$\epsilon(t) = \sigma_0 J_m(t) \quad (62)$$

where  $J_m(t)$  is the creep compliance of the material. The creep compliance is the strain per unit of applied stress, and describes uniquely the stress-strain behavior of a linearly viscoelastic material.

### 8.1 LAPLACE TRANSFORMATIONS

If  $\sigma_0$  is set equal to unity and the Laplace transform of Equation (62) is taken, the following result is obtained:

$$\bar{\epsilon}(s) = J_m(s) \quad (63)$$

where  $s$  is the Laplace parameter and the Laplace transform is defined by

$$\bar{f}(s) = \int_0^{\infty} f(t)e^{-st}dt \quad (64)$$

If a standard stress relaxation test is now considered, wherein a uniaxial displacement is applied resulting in a normal strain  $\epsilon_0$  which is held constant, the corresponding time-dependent uniaxial stress is related to the constant uniaxial strain through the relationship

$$\sigma(t) = \epsilon_0 E_m(t) \quad (65)$$

If  $\epsilon_0$  is set equal to unity and the Laplace transform of Equation (65) is taken, the following result is obtained:

$$\bar{\sigma}(s) = \bar{E}_m(s) \quad (66)$$

In the Laplace domain the "associated elastic" stress-strain relationship may be written

$$\bar{\sigma}(s) = E_m(s) \bar{\epsilon}(s) \quad (67)$$

and the corresponding strain-stress relationship may be written

$$\bar{\epsilon}(s) = J_m(s) \bar{\sigma}(s) \quad (68)$$

where  $\epsilon(s)$  and  $J(s)$  are the associated elastic modulus and compliance, respectively. If conditions of the standard uniaxial creep test, i.e.,

$$\epsilon(t) = J_m(t) \quad (69)$$

where

$$\sigma_o = 1 \quad (70)$$

are applied to Equation (68), the following results:

$$\bar{J}_m(s) = \frac{J_m(s)}{s} \quad (71)$$

or

$$J_m(s) = s\bar{J}_m(s) \quad (72)$$

If conditions of the standard uniaxial stress relaxation test, i.e.,

$$\sigma(t) = E_m(t) \quad (73)$$

where

$$\epsilon_o = 1 \quad (74)$$

are applied to Equation (67), the following results:

$$\bar{E}_m(s) = \frac{E_m(s)}{s} \quad (75)$$

or

$$E_m(s) = s\bar{E}_m(s) \quad (76)$$

Combining Equations (72) and (76) yields

$$\bar{E}_m(s) = \frac{1}{s^2 \bar{J}_m(s)} \quad (77)$$



since, from Equations (67) and (68),

$$E_m(s) = \frac{1}{J_m(s)} \quad (78)$$

From the definition of Poisson's ratio,  $\nu$ , as a function of time,

$$\nu(t) = - \frac{\epsilon_T(t)}{\epsilon_L(t)} \quad (79)$$

where  $\epsilon_T$  is transverse extensional strain and  $\epsilon_L$  is longitudinal extensional strain in a uniaxial test. If the uniaxial strain is held constant, Equation (79) becomes:

$$\nu(t) = - \frac{\epsilon_T(t)}{\epsilon_0} \quad (80)$$

Taking the Laplace transform of both sides of this equation results in the following:

$$\bar{\nu}(s) = - \frac{\bar{\epsilon}_T(s)}{\epsilon_0} \quad (81)$$

and therefore,

$$\epsilon_0 = - \frac{\bar{\epsilon}_T(s)}{\bar{\nu}(s)} \quad (82)$$

In the Laplace domain, Poisson's ratio is defined as:

$$\nu(s) = - \frac{\bar{\epsilon}_T(s)}{\bar{\epsilon}_L(s)} \quad (83)$$

With the assumed constant uniaxial strain,  $\epsilon_0$ ,  
Equation (83) becomes:

$$v(s) = - \frac{\bar{\epsilon}_T(s)}{\epsilon_0/s} \quad (84)$$

or

$$v(s) = - \frac{s\bar{\epsilon}_T(s)}{\epsilon_0} \quad (85)$$

and therefore,

$$\epsilon_0 = - \frac{s\bar{\epsilon}_T(s)}{v(s)} \quad (86)$$

Setting the right-hand sides of Equations (82) and (86)  
equal to each other, the following relationship is obtained:

$$- \frac{\bar{\epsilon}_T(s)}{\bar{v}(s)} = - s \frac{\bar{\epsilon}_T(s)}{v(s)} \quad (87)$$

This equation reduces to

$$\frac{1}{\bar{v}(s)} = \frac{s}{v(s)} \quad (88)$$

or

$$v(s) = s \bar{v}(s) \quad (89)$$

which is the "associated elastic" Poisson's ratio as a  
function of the Laplace parameter.

## 8.2 DETERMINATION OF CREEP COMPLIANCE, $J_m(t)$

The creep compliance of a linearly viscoelastic material  
is determined by a curve fit to experimental data obtained

from a standard creep test. If a least squares curve fit is utilized, the creep compliance  $J_m(t)$  may be expressed in a Prony series formulation as [16].

$$J_m(t) = D + \sum_{i=1}^m F_i \exp(-R_i t) + Ht \quad (90)$$

where  $D$ ,  $F_i$ , and  $H$  are constants evaluated from the least squares curve fit to experimental data and the  $E_i$  values are assigned.

### 8.3 DETERMINATION OF RELAXATION MODULUS, $E_m(t)$

The relaxation modulus and creep compliance are related by their Laplace transforms as given in Equation (78). By inverting this relationship through the use of partial fractions, the expression for the relaxation modulus,  $E_m(t)$  can be obtained.

The Laplace transform of the creep compliance in Equation (90) is:

$$\bar{J}_m(s) = \frac{D}{s} + \sum_{i=1}^m \frac{F_i}{s + R_i} + \frac{H}{s^2} \quad (91)$$

If  $m = 4$ , for example, the expanded form is :

$$\bar{J}_m(s) = \frac{D}{s} + \frac{F_1}{s+R_1} + \frac{F_2}{s+R_2} + \frac{F_3}{s+R_3} + \frac{F_4}{s+R_4} + \frac{H}{s^2} \quad (92)$$

which can be written as

$$\bar{J}_m(s) = \frac{C_1 s^5 + C_2 s^4 + C_3 s^3 + C_4 s^2 + C_5 s + C_6}{s^2 (s+R_1) (s+R_2) (s+R_3) (s+R_4)} \quad (93)$$

where the  $C_i$ ,  $i=1,2,\dots,6$ , are coefficients evaluated by taking a common denominator.

From the relationship given in Equation (77),

$$\bar{E}_m(s) = \frac{1}{s^2 J_m(s)} = \frac{(s+R_1)(s+R_2)(s+R_3)(s+R_4)}{C_1 s^5 + C_2 s^4 + C_3 s^3 + C_4 s^2 + C_5 s + C_6} \quad (94)$$

Solving for the roots of the fifth order polynomial in the denominator, and denoting the roots as  $\lambda_j$ , where  $j = 1, 2, \dots, 5$  yields the expression:

$$\bar{E}_m(s) = \frac{(s+R_1)(s+R_2)(s+R_3)(s+R_4)}{(s+\lambda_1)(s+\lambda_2)(s+\lambda_3)(s+\lambda_4)(s+\lambda_5)} \quad (95)$$

Inverting this equation by the method of partial fractions [17] gives the form

$$\bar{E}_m(s) = \frac{P(s)}{Q(s)} \quad (96)$$

where  $Q(s)$  and  $P(s)$  are polynomials and the degree of  $P(s)$  is less than the degree of  $Q(s)$ . Equation (95) may be written as the sum of partial fractions:

$$\bar{E}_m(s) = \frac{B_1}{s+\lambda_1} + \frac{B_2}{s+\lambda_2} + \frac{B_3}{s+\lambda_3} + \frac{B_4}{s+\lambda_4} + \frac{B_5}{s+\lambda_5} \quad (97)$$

where  $B_j$ ,  $j = 1, 2, \dots, 5$ , is evaluated:

$$B_j = \frac{P(\lambda_j)}{Q'(\lambda_j)} \quad (98)$$

The variable  $Q'(\lambda_j)$  is the first time derivative of the polynomial  $Q(\lambda_j)$  and  $P(\lambda_j)$  is the polynomial evaluated at the roots  $\lambda_j$ . The resulting equation for the relaxation modulus in the time domain is obtained:

$$E_m(t) = \sum_{j=1}^5 B_j \exp(-\lambda_j t) \quad (99)$$

#### 8.4 DETERMINATION OF POISSON'S RATIO, $\nu_m(t)$

The relationship of the bulk modulus  $K$  to the relaxation modulus and Poisson's ratio is employed:

$$K = \frac{E_m(t)}{3 [1 - 2\nu_m(t)]} \quad (100)$$

Therefore,

$$\nu_m(t) = \frac{3K - E_m(t)}{6K} \quad (101)$$

or

$$\nu_m(t) = \frac{1}{2} - \frac{E_m(t)}{6K} \quad (102)$$

The bulk modulus for the epoxy resin material system is taken as a constant. This assumption is consistent with experimental evidence and accepted practice. The time-dependent expressions for the matrix properties,  $E_m(t)$  and  $\nu_m(t)$  are thus determined.

## CHAPTER 9. ASSOCIATED ELASTIC SOLUTION

In order to study the time-dependent behavior of a filamentary composite spherical pressure vessel subjected to a constant internal pressure, the correspondence principle was employed. Based upon this principle, the problem is transformed from the time domain to the Laplace domain, and the time-dependent problem is replaced by an "associated elastic" problem with constitutive relationships and boundary conditions expressed as functions of the Laplace transform parameter  $s$  instead of the time parameter  $t$ . Inversion of the associated elastic solution then yields the solution for the time-dependent linearly viscoelastic problem.

The basic inversion process can usually be accomplished by the method of partial fractions if the mathematical model has a closed form elastic solution and simple material parameters [18]. However, for a typical actual material system, the function to be inverted is often known only for discrete positive real values of the transform parameter and, therefore, numerical techniques are appropriately used to obtain the transformed solution. Although numerical methods were not used to obtain the solution presented here, the complexity of the employed equations warranted the use of an appropriate numerical inversion technique. Among numerous techniques, the collocation method [19] is

apparently an accurate inversion technique which yields a relatively straightforward calculation, regardless of the complexity of material representation.

This approximate-inversion method yields a time-dependent solution of the form [20]:

$$p_f(t) = A + Bt + \sum_{v=1}^k h_v e^{-\frac{t}{\alpha_v}} \quad (103)$$

where  $p_f$  is burst pressure and  $A$ ,  $B$ ,  $h_v$ , and  $\alpha_v$  are constants. The constant  $A$  is evaluated from initial conditions,

$$A = p_f(t_0) - \sum_{v=1}^k h_v \quad (104)$$

In this case, an internal pressure  $p$  is applied at time  $t_0$  and held constant. Since the long-time value of  $p_f$  is assumed to be a finite constant, the value  $B = 0$  substituted into Equation (103) yields:

$$p_f(t) = A + \sum_{v=1}^k h_v e^{-\frac{t}{\alpha_v}} \quad (105)$$

Taking the Laplace transform of this equation and rearranging gives:

$$s \bar{p}_f(s) = A + s \sum_{v=1}^k \frac{h_v}{[(1/\alpha_v) + (1/\alpha_w)]} \quad (106)$$

where  $w = 1, 2, \dots, k$ .

Substituting Equation (104) into Equation (106) and rearranging yields:

$$\sum_{v=1}^k \frac{h_v}{(1 + \frac{\alpha_v}{\alpha_w})} = p_f(t_0) - s \bar{p}_f(s) \quad (107)$$

Because a six-term exponential function will be used to define the burst pressure  $p_f$  as a function of time, six discrete values of the Laplace parameter are utilized. An appropriate range of these values is chosen.

The selected expression for  $\alpha_v$  is [20]:

$$\alpha_v = e^{(7 - 2v)} \quad (v = 1, 2, \dots, 6) \quad (108)$$

and Equation (105) can be expressed as:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{1+e^{-2}} & \frac{1}{1+e^{-4}} & \frac{1}{1+e^{-6}} & \frac{1}{1+e^{-8}} & \frac{1}{1+e^{-10}} \\ \frac{1}{1+e^2} & \frac{1}{2} & \frac{1}{1+e^{-2}} & \frac{1}{1+e^{-4}} & \frac{1}{1+e^{-6}} & \frac{1}{1+e^{-8}} \\ \frac{1}{1+e^4} & \frac{1}{1+e^2} & \frac{1}{2} & \frac{1}{1+e^{-2}} & \frac{1}{1+e^{-4}} & \frac{1}{1+e^{-6}} \\ \frac{1}{1+e^6} & \frac{1}{1+e^4} & \frac{1}{1+e^2} & \frac{1}{2} & \frac{1}{1+e^{-2}} & \frac{1}{1+e^{-4}} \\ \frac{1}{1+e^8} & \frac{1}{1+e^6} & \frac{1}{1+e^4} & \frac{1}{1+e^2} & \frac{1}{2} & \frac{1}{1+e^{-2}} \\ \frac{1}{1+e^{10}} & \frac{1}{1+e^8} & \frac{1}{1+e^6} & \frac{1}{1+e^4} & \frac{1}{1+e^2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix} = \begin{bmatrix} p_f(t_0) - s \bar{p}_f(s) |_{s=e^{-5}} \\ p_f(t_0) - s \bar{p}_f(s) |_{s=e^{-3}} \\ p_f(t_0) - s \bar{p}_f(s) |_{s=e^{-1}} \\ p_f(t_0) - s \bar{p}_f(s) |_{s=e} \\ p_f(t_0) - s \bar{p}_f(s) |_{s=e^3} \\ p_f(t_0) - s \bar{p}_f(s) |_{s=e^5} \end{bmatrix} \quad (109)$$

Utilizing Equations (76) and (89) for the Laplace transforms of the relaxation modulus and Poisson's ratio, respectively, expressions for  $s \bar{E}_m(s)$  and  $s \bar{\nu}(s)$  are obtained.



These expressions are evaluated for discrete values of the Laplace parameter and inserted into the constitutive equations for the laminate given in Chapter 6. The laminate properties in the Laplace domain are then used to calculate burst pressure using the maximum strain theory of failure presented in Chapter 7. The associated elastic solution yielding  $\bar{p}_f(s)$  is thus obtained:

$$\bar{p}_f(s) = sK_\theta \frac{g(s)[\beta^{2n(s)} - 1]}{\frac{g(s)}{f(s)} + \beta^{2n(s)}} \quad (110)$$

The values of  $\bar{p}_f(s)$  and  $p_f(t_0)$ , the burst pressure at time zero, are substituted into Equations (107) from which the  $h_v$  values are computed. The value of A is then calculated from Equation (104). Substituting these values of  $h_v$  and A into Equation (105) yields the expression for the time-dependent burst pressure for the filamentary composite spherical pressure vessel.

A similar associated elastic solution may be used to calculate the critical strain,  $\epsilon_{\theta\theta}$ , as a function of time. The Laplace transform of a constant internal pressure is used to calculate  $s\bar{\epsilon}_{\theta\theta}(s)$ :

$$\epsilon_{\theta\theta}(s) = \frac{\bar{p}}{s} \frac{\frac{g(s)}{f(s)} + \beta^{2n(s)}}{g(s)[\beta^{2n(s)} - 1]} \quad (111)$$

The collocation method of inversion as outlined above is then used to obtain the time-dependent strain  $\epsilon_{\theta\theta}$  at the inner surface of the filamentary composite spherical pressure vessel.

The mathematical model for a linearly viscoelastic material therefore yields expressions which describe the time-dependent response of a filamentary composite structure. A numerical example will be presented to show the application of this method to a Kevlar/epoxy spherical pressure vessel subjected to a constant internal pressure.

## CHAPTER 10. APPLICATION

The material system chosen to demonstrate the analysis technique presented is a Kevlar/HBRF-55 filamentary composite. The HBRF-55 matrix material is a resin currently used in the filament wound motorcase of the Space Shuttle's Solid Rocket Boosters. The Kevlar fiber properties were supplied by the manufacturer and obtained from the literature [4] and are listed in Table 1. For the analysis, the fiber volume fractions (ratio of fiber volume to total laminate volume) used were 0.5, 0.6, and 0.7. These reflect values attainable for the system when wound into a spherical geometry. To demonstrate the effect of increasing the composite shell thickness, the solution was obtained for values of  $\beta$  (ratio of outer radius to inner radius) of 1.1, 1.3, and 1.5.

### 10.1 VISCOELASTIC MATERIAL PROPERTIES OF HBRF-55 RESIN

As described in Chapter 8, a creep (or equivalent) test is necessary in order to determine the time-dependent properties of a linearly viscoelastic material. Using the apparatus shown in Figure 4, a constant load was applied to a rectangular sample of HBRF-55 resin. The deformation of the sample was measured by a dial indicator, and at selected time intervals, the deformation of the sample was recorded.

TABLE 1. MATERIAL PROPERTIES

$$E_f = 19 \times 10^6 \text{ psi}$$

$$\nu_f = 0.2$$

Where the subscript f refers to the Kevlar fiber

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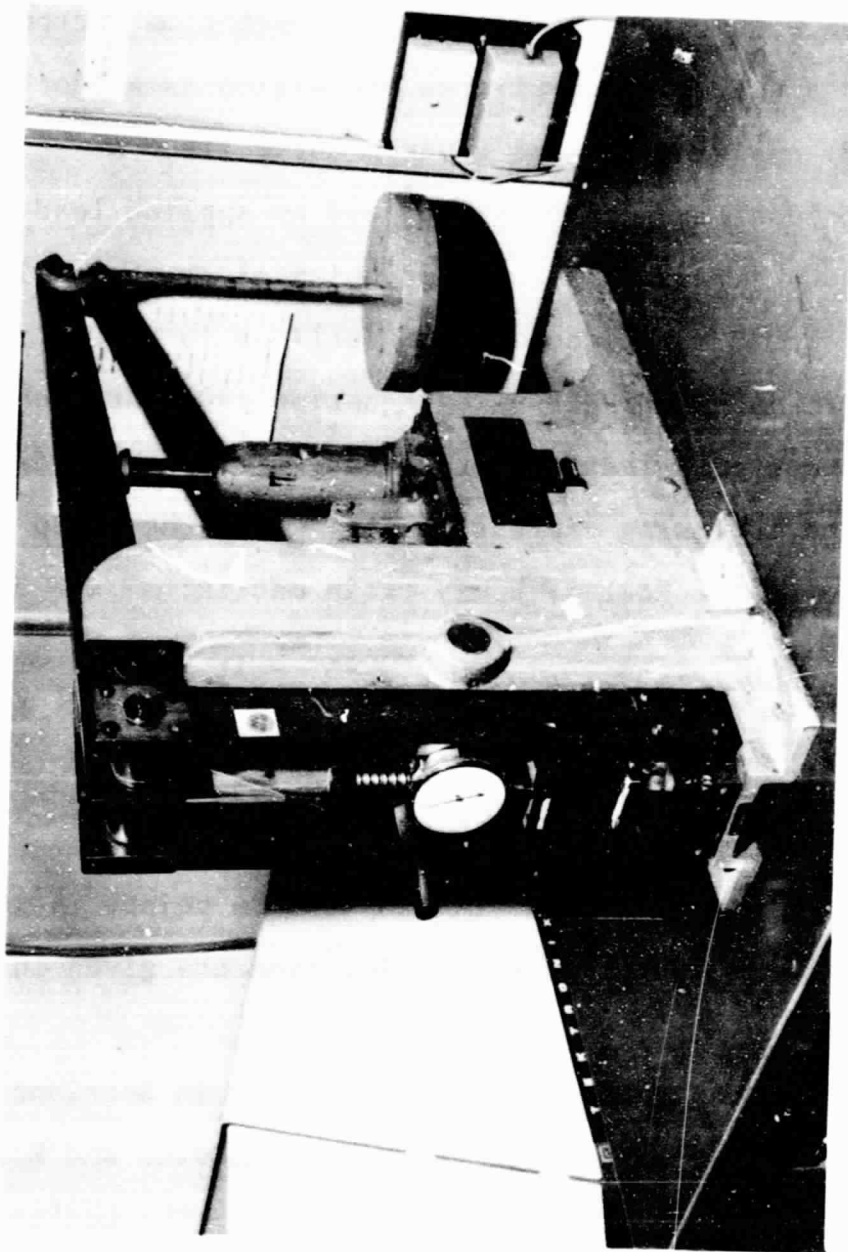


Figure 4. Experimental creep test apparatus.

The strain was calculated from Equation (111) and these values are listed in Table 2.

The FORTRAN program CREEP listed in Appendix A was written to calculate the creep compliance for each time increment and to obtain a creep compliance expression as a function of time by applying a least squares curve fit. A cross-sectional area of  $0.258 \text{ in}^2$  and an applied load of 107.9 pounds, provided a constant uniaxial stress of 574.2 psi. The creep compliance was, as outlined in Section 8.2, computed in program CREEP using Equation (90) for each time increment. A least squares curve fit of these data yielded the following expression for  $J_m(t)$ , the creep compliance of the HBRF-55 epoxy resin matrix:

$$J_m(t) = 0.2979 \times 10^{-5} + 0.2524 \times 10^{-6} e^{-.1t} \\ - 0.22 \times 10^{-6} e^{-t} - 0.5191 \times 10^{-7} e^{-10t} \\ - 0.181 \times 10^{-6} e^{-100t} + 0.223 \times 10^{-7} t \quad (112)$$

A plot of the curve fit and the input data points is given in Figure 5. These plots versus log time are given in Figure 6.

Following the development of equations in Sections 8.3 and 8.4, the expressions for relaxation modulus and Poisson's ratio as a function of time were obtained. The Laplace transform of  $J_m(t)$  is  $\bar{J}_m(s)$  and is programmed into CREEP. The expression for  $\bar{E}_m(s)$  is obtained from Equation (99) and a fifth order polynomial results in the denominator.

TABLE 2. EXPERIMENTAL CREEP TEST

Sample Length = 1.952 inches  
 Constant Applied Stress = 570.2 psi

<u>TIME</u> (hrs.)	<u>LOG TIME</u> (hrs.)	<u><math>\Delta L</math></u> (in. $\times 10^{-4}$ )	<u><math>\epsilon_c</math></u> (in/in $\times 10^{-3}$ )
0.001	-3.0	31.0	1.5881
0.001778	-2.75	31.5	1.6137
0.003167	-2.5	32.0	1.6393
0.005611	-2.25	32.2	1.6496
0.01	-2.0	32.6	1.6701
0.017778	-1.75	32.9	1.6855
0.031623	-1.5	33.2	1.7008
0.05556	-1.255	33.5	1.7162
0.1	-1.0	33.8	1.7316
0.1778	-0.75	34.0	1.7418
0.31611	-0.5	34.5	1.7674
0.5622	-0.25	34.8	1.7828
1.0	0.0	35.2	1.8033
1.945	0.289	35.9	1.8391
3.1622	0.5	36.2	1.8545
5.79	0.7627	36.3	1.8596
10.0	1.0	37.0	1.8955
17.7828	1.25	38.3	1.9621

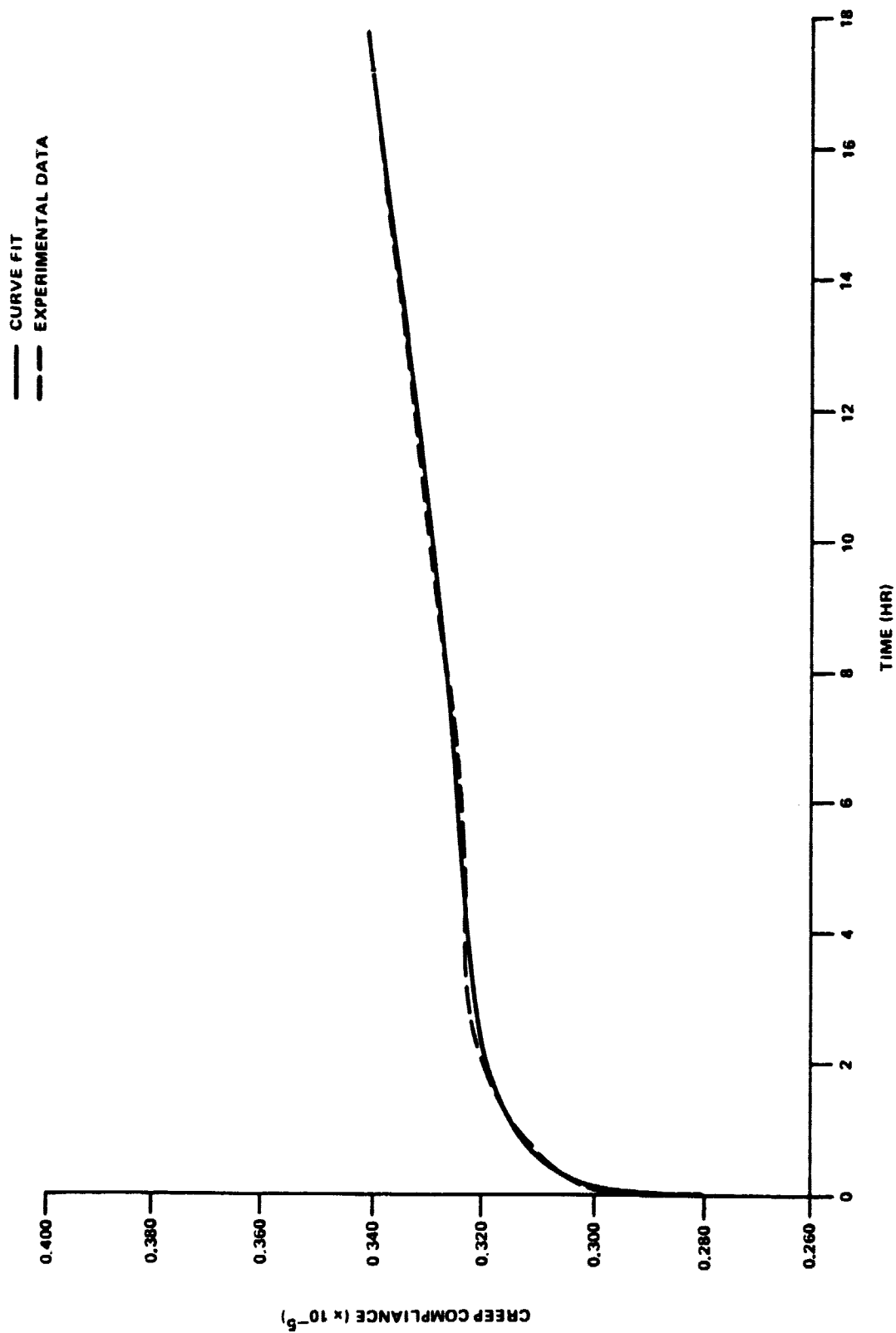


FIGURE 5. CREEP COMPLIANCE VS. TIME



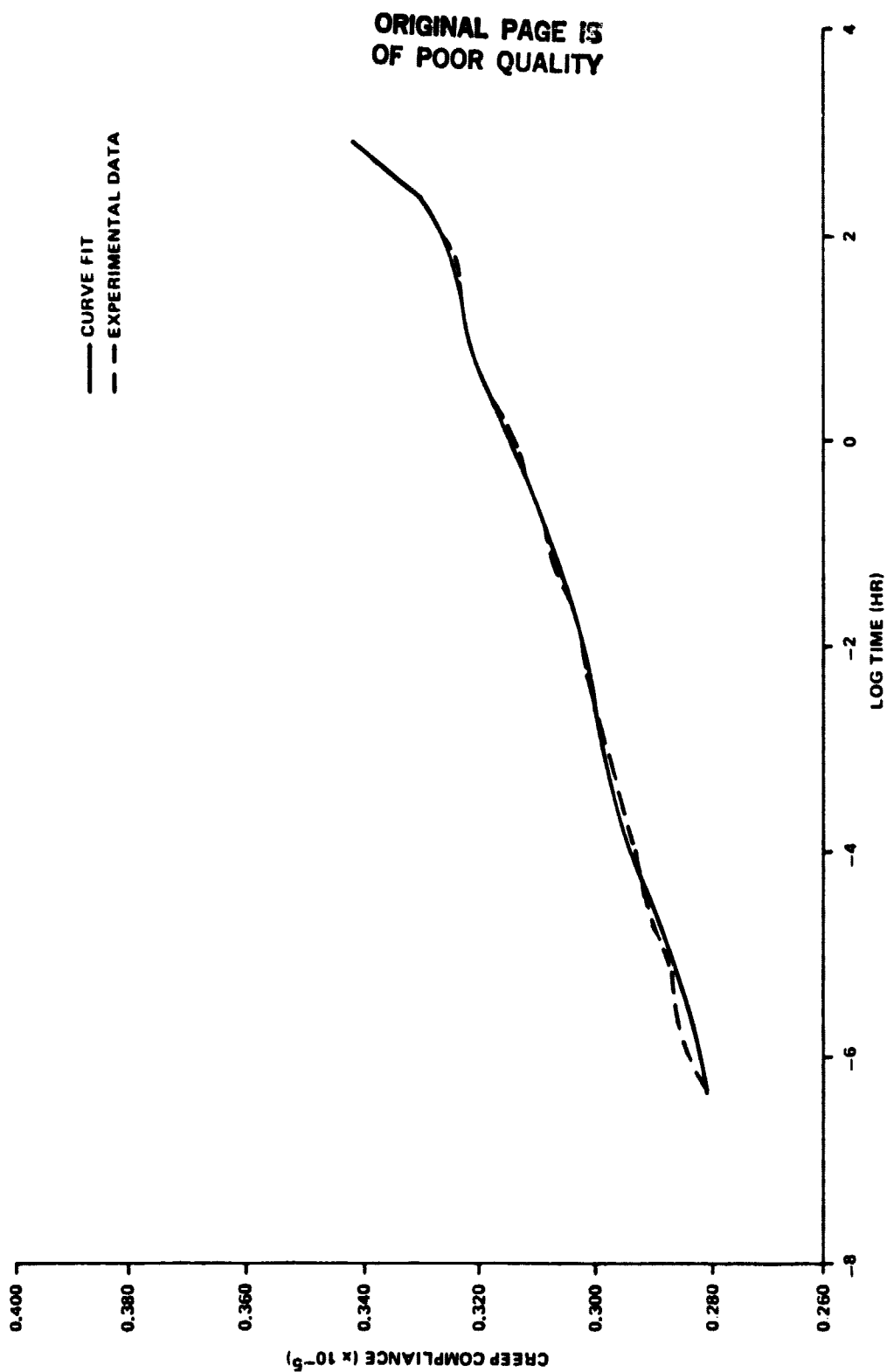


FIGURE 6. CREEP COMPLIANCE VS. LOG TIME

Roots of this polynomial were extracted using UNIVAC subroutine RTPOLY. The method of partial fractions is utilized in CREEP to invert the Laplace expression  $\bar{E}_m(s)$  into the time domain. The relaxation modulus,  $E_m(t)$ , is thus obtained for the epoxy resin HBRF-55:

$$E_m(t) = 0.2212 \times 10^5 e^{-1065t} + 0.5835 \times 10^4 e^{-10.18t} \\ + 0.2242 \times 10^5 e^{-1.07t} - 0.3064 \times 10^5 e^{-.092t} \\ + 0.3402 \times 10^6 e^{-.0075t} \quad (113)$$

A plot of  $E_m(t)$  versus log time is given in Figure 7. The relaxation modulus of HBRF-55 at time zero was calculated to be 360,000 psi.

The expression for Poisson's ratio,  $\nu_m(t)$ , was obtained using the expression above for the relaxation modulus,  $E_m(t)$ . Substituting into Equation (102) a bulk modulus for HBRF-55 of 400,000 psi and rearranging, the equation for the Poisson's ratio is:

$$\nu_m(t) = \frac{1}{2} - \frac{E_m(t)}{2,400,000} \quad (114)$$

From the expression for  $E_m(t)$  given in Equation (113) the resulting expression for  $\nu_m(t)$  is:

$$\nu_m(t) = -9.2167 \times 10^{-3} e^{-106.5t} - 1.4313 \times 10^{-3} e^{-10.18t} \\ - 9.3417 \times 10^{-3} e^{-2.07t} + 1.2767 \times 10^{-2} e^{-.092t} \\ - 1.4275 \times 10^{-1} e^{-.0075t} + 0.5 \quad (115)$$

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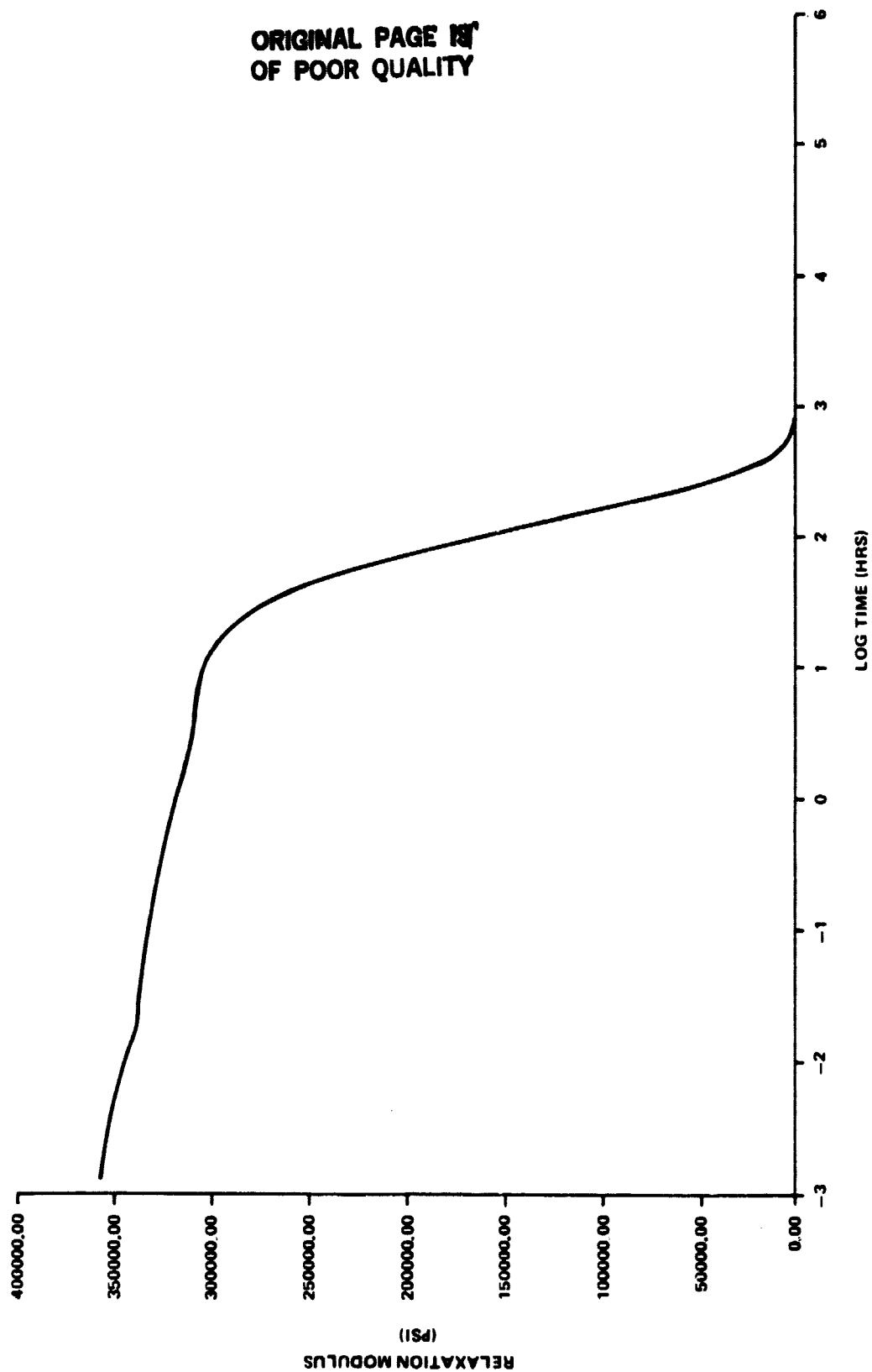


FIGURE 7. RELAXATION MODULUS VS. LOG TIME

A plot of  $v_m(t)$  versus log time is shown in Figure 8. The Poisson's ratio for HBRF-55 at time zero was calculated to be 0.35.

## 10.2 ASSOCIATED ELASTIC SOLUTION

The correspondence principle, along with the collocation method, as derived in Chapter 9, may be used with the maximum strain theory of failure developed in Chapter 7 to predict the burst pressure as a function of time for a filamentary composite spherical pressure vessel subjected to a constant internal pressure. A FORTRAN program, THESIS, was developed to generate the time-dependent response of a Kevlar/HBRF-55 system and is listed in Appendix B. The program predicts burst pressure and critical strain as a function of time.

### 10.2.1 PROPERTIES OF THE LAMINATE

The previously derived expressions for  $E_m(t)$  and  $v_m(t)$  are transformed into the Laplace domain utilizing Equations (76) and (89) and result in the equations:

$$s\bar{E}_m(s) = \frac{22,120 s}{s + 106.5} + \frac{5,835 s}{s + 10.18} + \frac{22,420 s}{s + 1.07} - \frac{30,640 s}{s + .092} + \frac{340,200 s}{s + .0075} \quad (116)$$

$$s\bar{v}_m(s) = 0.5 - \frac{0.0092 s}{s + 106.5} - \frac{0.0024 s}{s + 10.18} - \frac{0.0093 s}{s + 1.07} + \frac{0.0128 s}{s + .092} - \frac{0.1418 s}{s + .0075} \quad (117)$$

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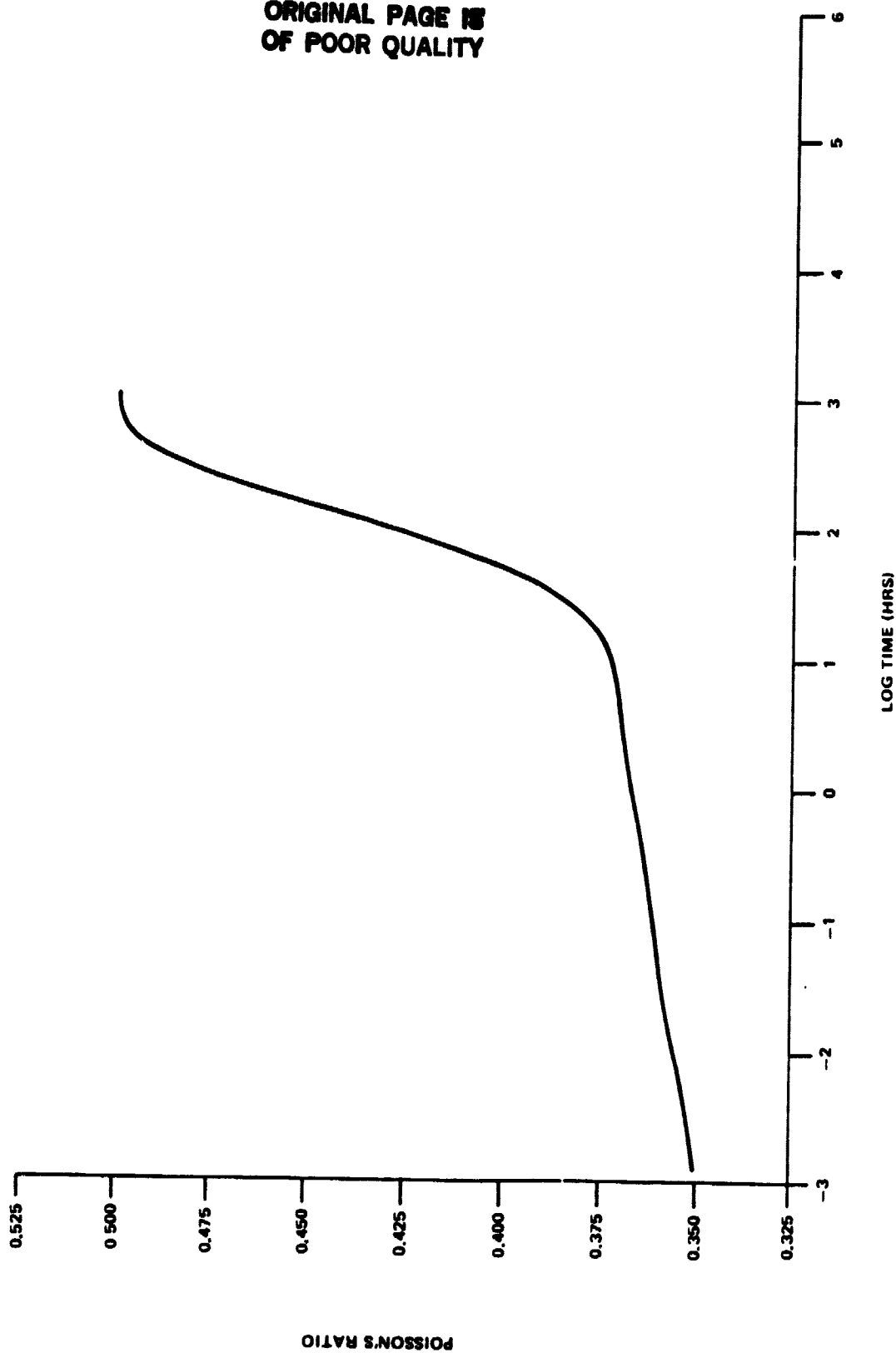


FIGURE 8. POISSON'S RATIO VS. LOG TIME

The matrix properties  $\nu_m$  and  $E_m$  are thus calculated in the program THESIS and summarized in Table 3. These values are then substituted into the lamina and laminate equations developed in Chapters 4, 5, and 6. For the case of  $\nu_f = 0.6$  and  $\beta = 1.1$ , lamina properties computed in THESIS as a function of the six values of the Laplace parameter are given in Table 4. Laminate properties are listed in Table 5. Also listed are the computed values for  $n$ , the degree of anisotropy. For an isotropic material system,  $n = 1.5$ , and the departure from this value indicates the degree of anisotropy in the system.

#### 10.2.2 MAXIMUM STRAIN THEORY OF FAILURE

The burst pressure and critical strain, utilizing Equations (61) and (59), respectively, are based upon the maximum strain theory of failure. In the Laplace domain, these expressions are given by Equations (110) and (111). The ultimate circumferential strain for Kevlar fiber is 0.0132 [4] and is the numerical value for  $K_\theta$ .

Equation (105) in the form of Equation (109) is then programmed into THESIS. The vector  $\{p_f(t_0) - s\bar{p}_f(s)\}$  is obtained by calculating  $p_f(t_0)$  from Equation (61) in program THESIS.CCC, given in Appendix B. The value for  $s\bar{p}_f(s)$  is subtracted from  $p_f(t_0)$  and evaluated for each discrete value of the Laplace parameter. From Equation

TABLE 3. VALUES OF  $s$ ,  $E_m(s)$ , and  $v_m(s)$ 

<u><math>\ln s</math></u>	<u><math>s</math></u>	<u><math>E_m(s)</math></u>	<u><math>v_m(s)</math></u>
5	148.4	350.2	0.3542
3	20.09	338.2	0.3591
1	2.718	327.5	0.3636
-1	0.368	314.9	0.3688
-3	0.050	286.1	0.3808
-5	0.0067	158.6	0.4339

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TABLE 4. VALUES FOR LAMINA PROPERTIES

 $(\nu_f = 0.6, \beta = 1.1)$ 

<u>Ln s</u>	<u>E<sub>L</sub>(s)</u>	<u>E<sub>T</sub>(s)</u>	<u>G<sub>LT</sub>(s)</u>	<u><math>\nu_{LT}</math>(s)</u>	<u><math>\nu_{TT}</math>(s)</u>
5	11.54	0.8519	0.5473	0.2617	0.3374
3	11.54	0.8236	0.5283	0.2636	0.3422
1	11.53	0.7981	0.5112	0.2654	0.3465
-1	11.53	0.7682	0.4913	0.2675	0.3517
-3	11.51	0.6994	0.4456	0.2723	0.3640
-5	11.46	0.3916	0.2453	0.2936	0.4251

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TABLE 5. VALUES FOR LAMINATE PROPERTIES AND  $n$   
 $(v_f = 0.6, \beta = 1.1)$

<u><math>\ln s</math></u>	<u><math>n(s)</math></u>	<u><math>C_{rr}(s)</math></u>	<u><math>C_{\theta\theta}(s)</math></u>	<u><math>C_{\theta\phi}(s)</math></u>	<u><math>C_{\theta r}(s)</math></u>	<u><math>E_{\theta}(s)</math></u>	<u><math>E_r(s)</math></u>
5	3.316	0.9712	5.118	1.569	0.3379	4.579	0.9371
3	3.356	0.9424	5.093	1.569	0.3319	4.553	0.9093
1	3.395	0.9163	5.071	1.569	0.3263	4.529	0.8842
-1	3.442	0.8855	5.045	1.569	0.3194	4.501	0.8546
-3	3.564	0.8141	4.984	1.567	0.3026	4.437	0.7862
-5	4.469	0.4814	4.697	1.537	0.2043	4.154	0.4680

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(109), the values for  $h_v$  are calculated in double precision with UNIVAC subroutine DPINV.

The constant A is evaluated from Equation (104), using the initial burst pressure and the sum of the values of  $h_v$ . This operation is easily accomplished in the program THESIS.

Finally, the burst pressure in the time domain can be calculated from Equation (110), where the units of time is hours. The burst pressures computed from THESIS are plotted with respect to log time. Figure 9 shows burst pressure as a function of fiber volume fraction,  $v_f$ . Figure 10 demonstrates the effect of varying  $\beta$ , the ratio of composite shell outer radius to inner radius.

In a similar fashion, the vector  $\{\epsilon_{\theta\theta}(t_0) - s\bar{\epsilon}_{\theta\theta}(s)\}$  is obtained by calculating initial strain  $\epsilon(t_0)$  in program THESIS.CCC. The value for  $s\bar{\epsilon}_{\theta\theta}(s)$  is subtracted from  $\epsilon_{\theta\theta}(t_0)$  and evaluated for each discrete value of the Laplace parameter. Then, the values of  $h_v'$  are calculated. The constant  $A'$  is evaluated and input into the time-dependent expression for strain. Computed values for strain from the program THESIS are plotted with respect to log time. Figure 11 is a plot of strain versus log time and illustrates the effect of fiber volume fraction,  $v_f$ . Figure 12 demonstrates the effect of varying  $\beta$  from 1.1 to 1.5.

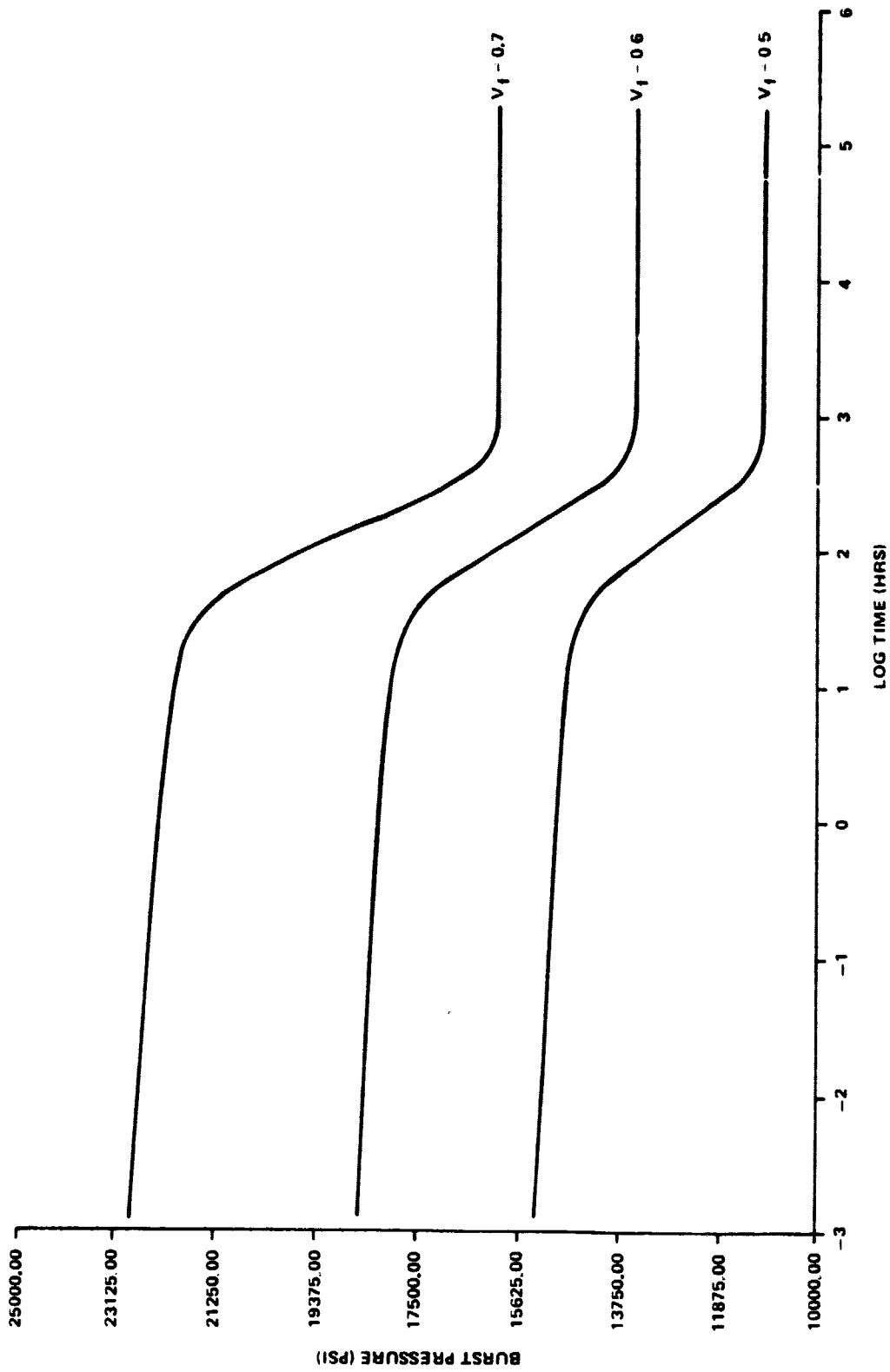


FIGURE 9. BURST PRESSURE VS. LOG TIME, VARYING FIBER VOLUME FRACTION,  $V_f$

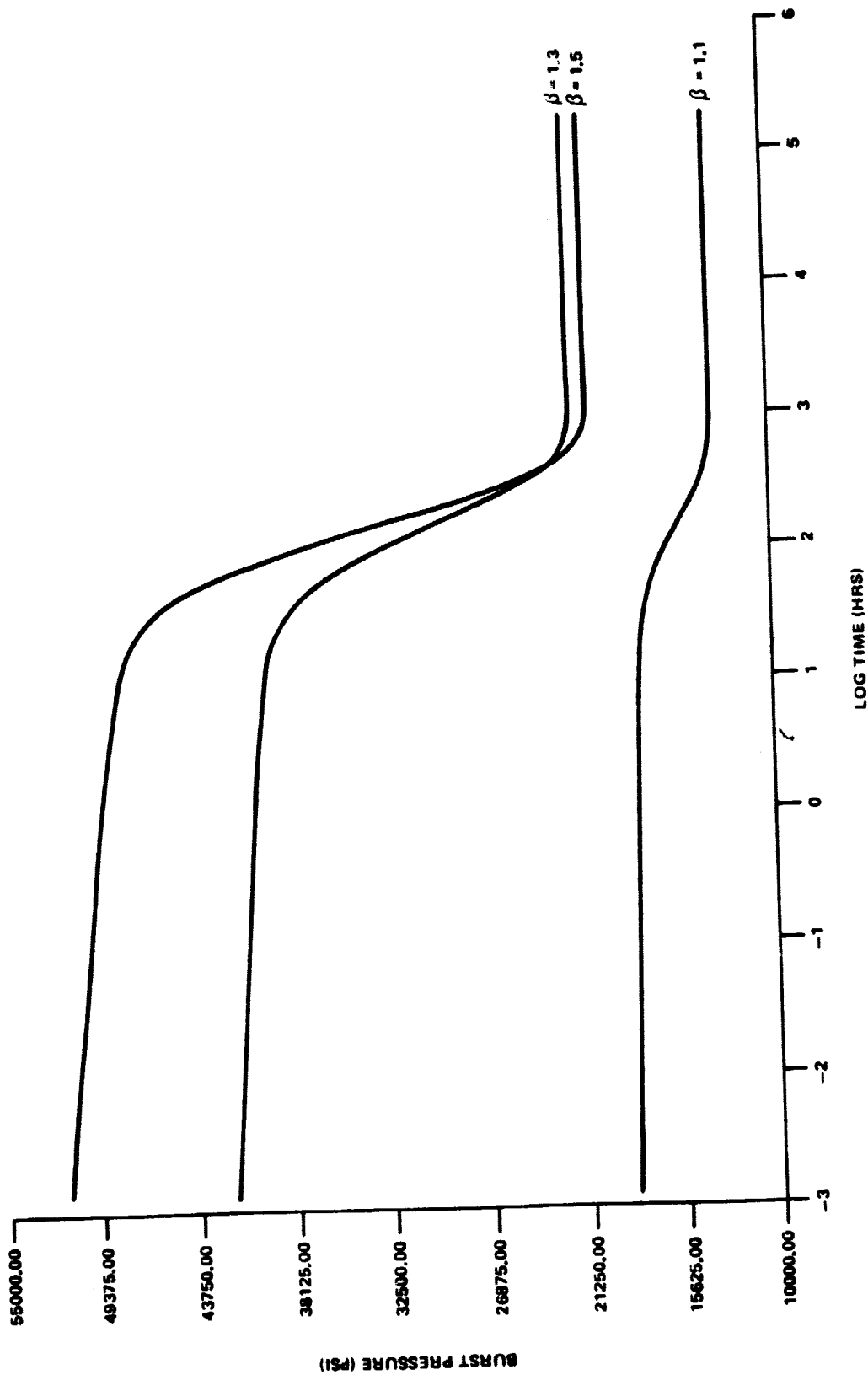


FIGURE 10. BURST PRESSURE VS. LOG TIME, VARYING  $\beta$  ( $= r_o/r_i$ )

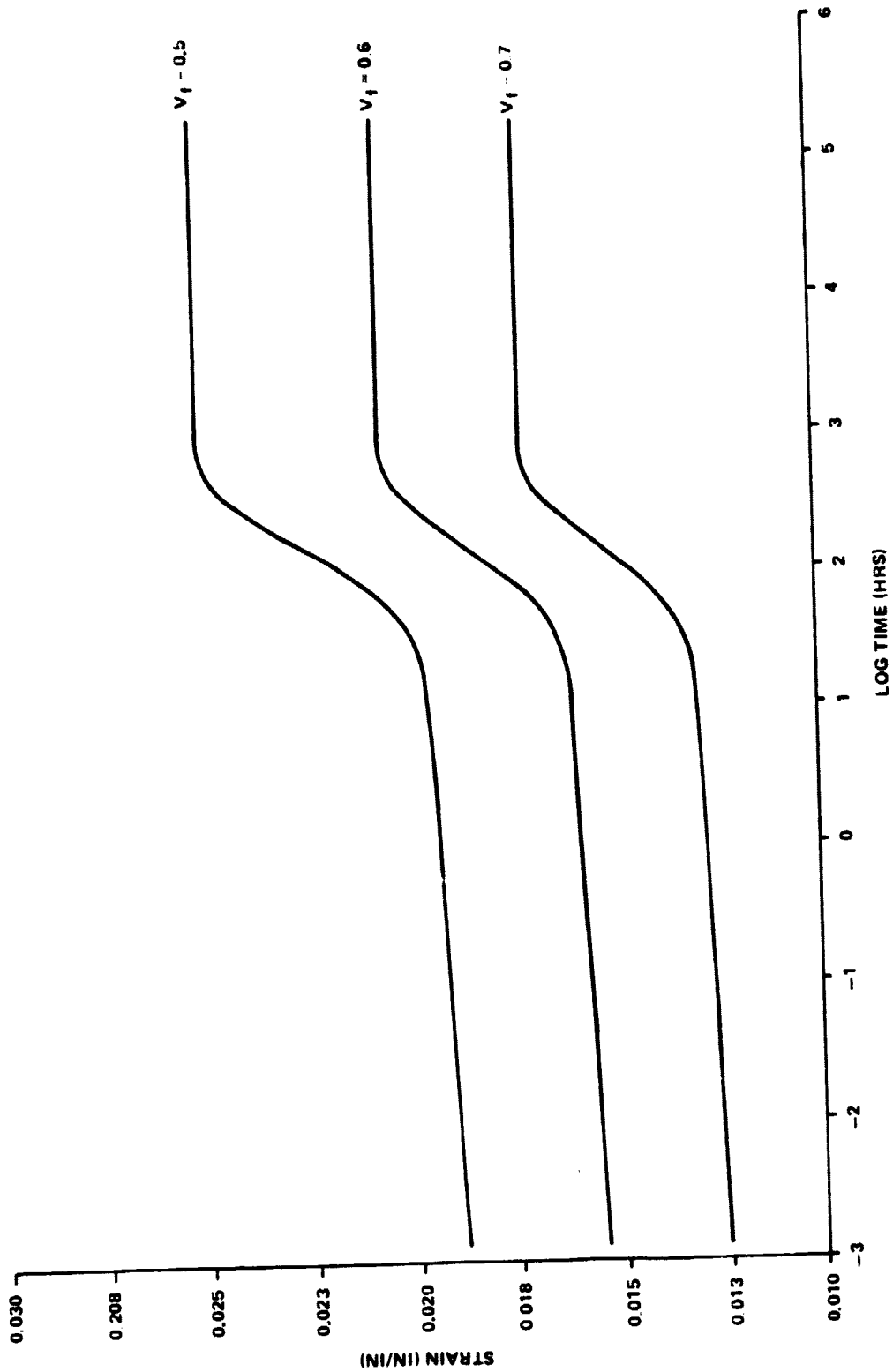


FIGURE 11. STRAIN VS. LOG TIME, VARYING FIBER VOLUME FRACTION,  $V_f$

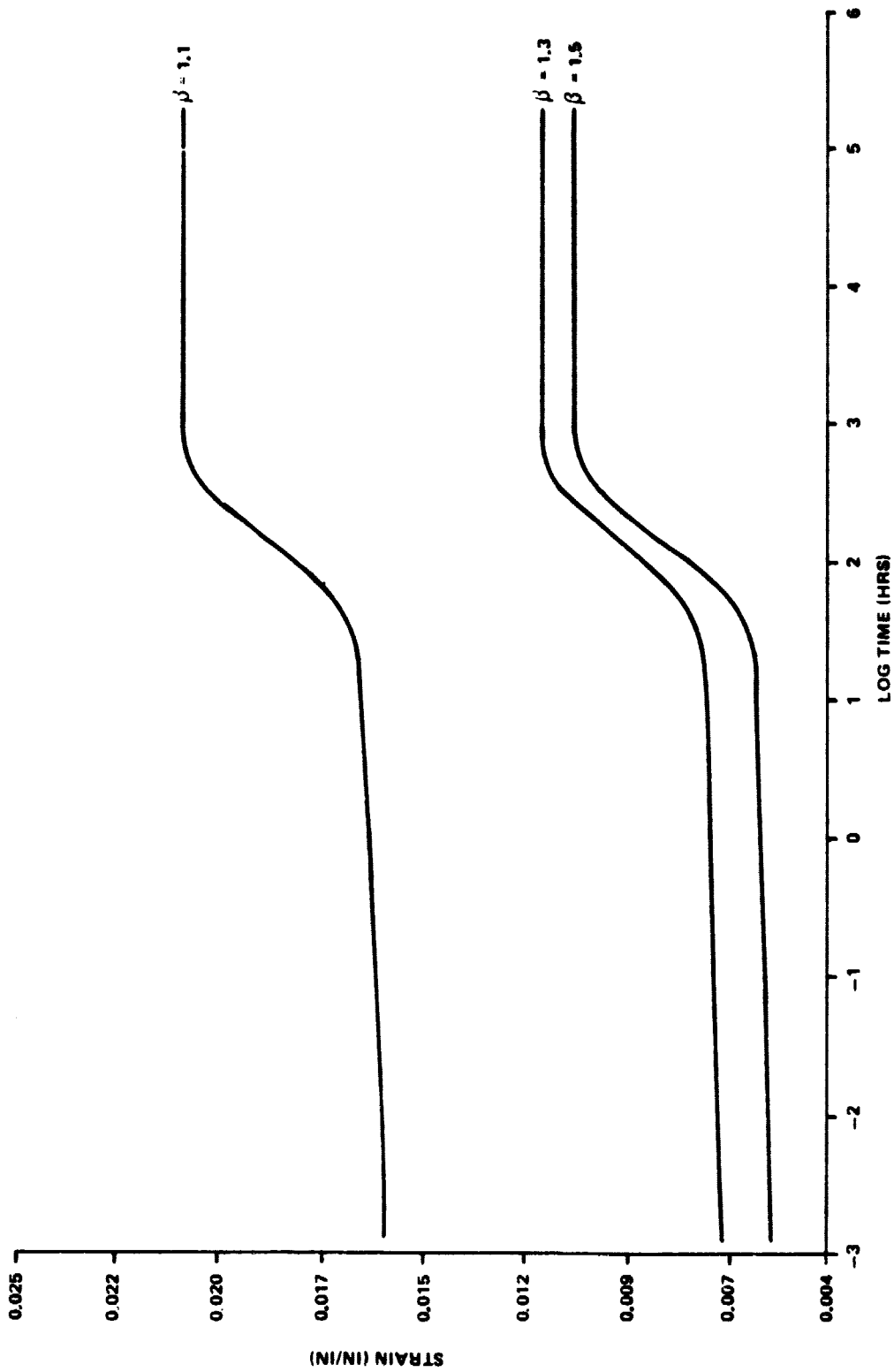


FIGURE 12. STRAIN VS. LOG TIME, VARYING  $\beta$  ( $= r_o/r_i$ )

A comparison of predicted burst pressure in Figure 9 and strain in Figure 11 is shown for fiber volume fractions 0.5, 0.6, and 0.7. These plots show that an increase in fiber volume fraction increases the performance of the composite. In particular, Figure 11 shows that the failure strain of Kevlar, 0.0192, is approached almost immediately for  $v_f = 0.5$ , is approached much later for  $v_f = 0.6$ , and is never approached for  $v_f = 0.7$ .

The effect of shell thickness on vessel performance is shown by varying  $\beta$  from 1.1 to 1.5 in Figures 10 and 12.

Figures 9 through 12 summarize the results of THESIS, a program of the time-dependent burst pressure and strain equations from the associated elastic solution based upon the maximum strain theory.

## CHAPTER 11. CONCLUSIONS

The maximum strain theory of failure developed for composite filamentary pressure vessels, when combined with the linearly viscoelastic behavior of the composite matrix, yields a solution which predicts burst pressure and critical strain for the vessels as a function of time. This solution is obtained by solving the equations in the Laplace domain and inverting them back into the time domain using the method of collocation. An evaluation of the burst pressures and strains computed using this method demonstrates that when the time-dependent response of a composite matrix is considered, the failure performance of the vessel is reduced. It is thus concluded that this general method may be used to predict a time-dependent response, and that the linearly viscoelastic properties of a composite matrix contribute to this reduction in vessel performance. Knowing the relaxation modulus and Poisson's ratio of a matrix with respect to time, design criteria for a filamentary composite spherical pressure vessel could be derived which consider the time-dependent response of the matrix.

The numerical evaluation of the Kevlar-HBRF-55 system illustrates the reduction in burst pressure and increase in critical strain with respect to time. The inclusion of the viscoelastic material properties of the resin into the associated elastic solution is thus a viable approach for predicting time-dependent vessel performance.



There are additional mechanisms which affect vessel performance in time, such as nonlinear effects and/or matrix crazing, perhaps initiated by this modeled time-dependent matrix stiffness reduction. The same type analysis could include a damage criterion or function and provide a more accurate prediction of time-dependent response. It has been shown that this type of analysis can be used to predict the time-dependent response of filamentary composite spherical pressure vessels and can possibly be used to identify the mechanism(s) which cause the observed failures.

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**APPENDIXES**

**APPENDIX A****FORTRAN PROGRAM CREEP**

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**&**

119	270	formation roots of an n-in order polynomial	
120		HEAL36,233H11	
121	230	formation the given is n = 12777	
122		PRINT 200	
123	240	formation the polynomial coefficients are 0	
124		HEAL36,233H11COEFF(1:12777)	
125	210	formation 0	
126		PRINT 200	
127	240	formation 100, -real roots	
128		HEAL36,233H11COEFF(1:12777)	
129	270	formation 20, 0	
130		PRINT 200	
131	240	formation 100, -real roots	
132		HEAL36,233H11COEFF(1:12777)	
133	270	formation 20, 0	
134		PRINT 200	
135	240	formation 100, -real roots	
136		HEAL36,233H11COEFF(1:12777)	
137	270	formation 20, 0	
138		PRINT 200	
139	240	formation 100, -real roots	
140		HEAL36,233H11COEFF(1:12777)	
141	270	formation 20, 0	
142		PRINT 200	
143	240	formation 100, -real roots	
144		HEAL36,233H11COEFF(1:12777)	
145	270	formation 20, 0	
146		PRINT 200	
147	240	formation 100, -real roots	
148		HEAL36,233H11COEFF(1:12777)	
149	270	formation 20, 0	
150		PRINT 200	
151	240	formation 100, -real roots	
152		HEAL36,233H11COEFF(1:12777)	
153	270	formation 20, 0	
154		PRINT 200	
155	240	formation 100, -real roots	
156		HEAL36,233H11COEFF(1:12777)	
157	270	formation 20, 0	
158		PRINT 200	
159	240	formation 100, -real roots	
160		HEAL36,233H11COEFF(1:12777)	
161	270	formation 20, 0	
162		PRINT 200	
163	240	formation 100, -real roots	
164		HEAL36,233H11COEFF(1:12777)	
165	270	formation 20, 0	
166		PRINT 200	
167	240	formation 100, -real roots	
168		HEAL36,233H11COEFF(1:12777)	
169	270	formation 20, 0	
170		PRINT 200	



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171	5622	1.78281-03
172	1	1.80331-03
173	1.905	1.83911-03
174	3.1822	1.85451-03
175	5.79	1.85961-03
176	10	1.89551-03
177	17.7828	1.96211-03

INITIAL STRESS 55.574,2000

INPUT DATA POINTS

TIME (HRS)	STRAIN	COMPLANCE
.001000	.1500-002	.2765-005
.001778	.1618-002	.2810-005
.003167	.1639-002	.2855-005
.005611	.1650-002	.2873-005
.010000	.1670-002	.2909-005
.017778	.1685-002	.2935-005
.031623	.1701-002	.2962-005
.055560	.1716-002	.2989-005
.100000	.1732-002	.3016-005
.177800	.1742-002	.3033-005
.316110	.1767-002	.3078-005
.562200	.1783-002	.3105-005
1.000000	.1803-002	.3141-005
1.945000	.1839-002	.3203-005
3.162200	.1855-002	.3230-005
5.179000	.1860-002	.3239-005
10.000000	.1895-002	.3301-005
17.782800	.1952-002	.3317-005

INPUT A MATRIX

.1000\*002 .1543\*002 .1197\*002 .7519\*001 .3624\*001 .9096\*002  
 .1543\*002 .1426\*002 .1131\*002 .7497\*001 .3623\*001 .1598\*002  
 .1197\*002 .1131\*002 .1022\*002 .1350\*001 .3600\*001 .1110\*001  
 .7519\*001 .7497\*001 .6323\*001 .3471\*001 .1123\*000  
 .3624\*001 .3623\*001 .3600\*001 .3471\*001 .2542\*001 .1614\*001  
 .9096\*002 .1598\*002 .1110\*001 .1123\*000 .1614\*001 .9050\*003

INPUT B MATRIX

.5466-004 .4610-004 .5377-004 .2171-004 .1028-004 .1360-003

SOLUTION 2 VECTOR

.2979-005 .2529-006 .2200-006 .5191-007 .1810-006 .2230-007

FINAL SOLUTION-CURVE FIT

TIME	LOG TIME	COMPLIANCE
-0.01000	-6.907755	-2197-005
-0.01178	-6.332266	-2809-005
-0.01167	-5.754970	-2830-005
-0.05611	-5.193026	-2860-005
-0.01000	-4.605170	-2900-005
-0.17778	-4.029794	-2941-005
-0.31623	-3.453871	-2973-005
-0.55560	-2.890292	-3013-005
-1.00000	-2.302585	-3013-005
-1.71800	-1.727096	-3038-005
-3.16110	-1.151665	-3068-005
-5.62200	-0.575898	-3105-005
-1.00000	-0.000000	-3149-005
1.985000	-665.662	-3199-005
3.172300	1.151728	-3228-005
5.190000	1.756132	-3249-005
10.000000	2.302585	-3295-005
17.782000	2.878232	-3416-005

ROOTS OF AN N-TH ORDER POLYNOMIAL  
THE ORDER IS N = 5

THE POLYNOMIAL COEFFICIENTS ARE

.2778562-005 .52753151-003 .33923092-002 .35644658-002 .32268374-003 .22299372-005

REAL ROOTS

-.75329294-002  
-.91617210-001  
-.10727475-001  
-.10175597-002  
-.10653031-003

ROOT

POLYNOMIAL

F(1)

G(1)

A(1)

-.75329294-002 .33087225-023 .71694485-002 .26955892-003 .44164448-004  
-.91617210-001 -.52395559-022 .75381051-001 -.24603340-003 .30-18544-005  
-.10727475-001 -.71997801-020 .62495981-002 .27076609-002 .22910789-005  
-.10175597-002 .53922939-017 -.18582003-004 -.24988584-000 .58-1.660-004  
-.10653031-003 .14771279-013 .70801066-007 .32012066-003 .22316993-005

1 AT 2100 .3599-006

PRINT, S, LLC  
FOR PAPER 200112 136 514111 04/23/83 15:53:29

54

## APPENDIX B

## FORTRAN PROGRAM THESIS

11

[illegible]

11



```

171 A14,11=1./11.,*OFX(10,1)
172 A15,21=1./11.,*OFX(10,1)
173 A16,31=1./11.,*OFX(10,1)
174 A17,41=1./11.,*OFX(10,1)
175 A18,51=1./11.,*OFX(10,1)
176 A19,61=1./11.,*OFX(10,1)
177 A20,71=1./11.,*OFX(10,1)
178 A21,81=1./11.,*OFX(10,1)
179 A22,91=1./11.,*OFX(10,1)
180 A23,101=1./11.,*OFX(10,1)
181 A24,111=1./11.,*OFX(10,1)
182 A25,121=1./11.,*OFX(10,1)
183 A26,131=1./11.,*OFX(10,1)
184 A27,141=1./11.,*OFX(10,1)
185 A28,151=1./11.,*OFX(10,1)
186 A29,161=1./11.,*OFX(10,1)
187 A30,171=1./11.,*OFX(10,1)
188 A31,181=1./11.,*OFX(10,1)
189 A32,191=1./11.,*OFX(10,1)
190 A33,201=1./11.,*OFX(10,1)
191 A34,211=1./11.,*OFX(10,1)
192 A35,221=1./11.,*OFX(10,1)
193 A36,231=1./11.,*OFX(10,1)
194 A37,241=1./11.,*OFX(10,1)
195 A38,251=1./11.,*OFX(10,1)
196 A39,261=1./11.,*OFX(10,1)
197 A40,271=1./11.,*OFX(10,1)
198 A41,281=1./11.,*OFX(10,1)
199 A42,291=1./11.,*OFX(10,1)
200 A43,301=1./11.,*OFX(10,1)
201 A44,311=1./11.,*OFX(10,1)
202 A45,321=1./11.,*OFX(10,1)
203 A46,331=1./11.,*OFX(10,1)
204 A47,341=1./11.,*OFX(10,1)
205 A48,351=1./11.,*OFX(10,1)
206 A49,361=1./11.,*OFX(10,1)
207 A50,371=1./11.,*OFX(10,1)
208 A51,381=1./11.,*OFX(10,1)
209 A52,391=1./11.,*OFX(10,1)
210 A53,401=1./11.,*OFX(10,1)
211 A54,411=1./11.,*OFX(10,1)
212 A55,421=1./11.,*OFX(10,1)
213 A56,431=1./11.,*OFX(10,1)
214 A57,441=1./11.,*OFX(10,1)
215 A58,451=1./11.,*OFX(10,1)
216 A59,461=1./11.,*OFX(10,1)
217 A60,471=1./11.,*OFX(10,1)
218 A61,481=1./11.,*OFX(10,1)
219 A62,491=1./11.,*OFX(10,1)
220 A63,501=1./11.,*OFX(10,1)
221 A64,511=1./11.,*OFX(10,1)
222 A65,521=1./11.,*OFX(10,1)
223 A66,531=1./11.,*OFX(10,1)
224 A67,541=1./11.,*OFX(10,1)
225 A68,551=1./11.,*OFX(10,1)
226 A69,561=1./11.,*OFX(10,1)
227 A70,571=1./11.,*OFX(10,1)

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228 HV111=AA11,71
229 SUMSIS=SUMSIS+GV11
230 SUMSIN=SUMSIN+HV11
231 CONTINUE
232 3000
233 WRITE(6,170)
234 170 FORMAT(//,3X,'SOLUTION G MATRIX',//)
235 WRITE(6,120)GV11,1,1,61
236 WRITE(6,180)SUMSIS
237 180 FORMAT(//,5X,'SUM OF GV=',D12.4)
238 C
239 C EVALUATE CONSTANT A FROM INITIAL STRAIN AND STRESS
240 C CONDITIONS
241 C
242 AC=BP1-SUMSIS
243 WRITE(6,190)AC
244 190 FORMAT(//,5X,'A CONSTANT=',D12.4)
245 WRITE(6,210)
246 210 FORMAT(//,33X,'SOLUTION H MATRIX',//)
247 WRITE(6,120)HV11,1,1,61
248 220
249 220
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205	IN MAIN/MIH			
206	LIB SYSTEMS CO.			
207	ATOF			
208	2801 MEN			
209	19000000.	2		
210	490	1.1	0.192	15000.
211	00647	.05		.168
212	0.5	0.5		2.718
213	0.6			20.09
214	0.4			100.0
215	0.7			
216	0.3			
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CASE 1  
\*\*\*\*\*

L.F. = .1900+008

POISSON, F. = .2000+000

RE TA = .1100+001

N. TH. TA = .1920+001

V.S. = .5000+000

V.M. = .5000+000

INITIAL UNST PRESSURE = .1531+005

INITIAL STRAIN = .1881+001

POISSON, M = .4319+000

C, I = .7917+007

C, M = .5530+005

C, L = .9519+007

C, T = .3165+006

POISSON, L I = .3170+000

POISSON, L L = .1091+001

POISSON, L T = .4808+000

C, L I = .1710+006

C, M = .1546+006

C, L L = .9703+007

C, L T = .4130+006

C, T I = .1995+006

C, T L = .2006+006

C, T T = .1062+006

LAMBDA = .1317+001

C, M I = .4130+006

C, M L = .1921+007

C, M T = .3325+007

C, M M = .1975+006

C, M L = .3438+007

C, M T = .1961+006

N = .4408+001

F = .2009+007

G = .1632+007

BURST PRESSURE = .1119+005

STRAIN = .3260+001

POISSON, R = .1808+000

G, T = .7917+007

G, R = .1016+006

L, T = .9692+007

L, R = .5637+006

POISSON, L T = .2904+000

POISSON, R T = .1697+001

POISSON, L R = .4094+000

G, L T = .3145+006

L, R = .2861+006

G, L R = .9805+007

G, R T = .6818+006

L, L T = .2799+006

G, R R = .2804+006

G, L R = .2607+006

LAMBDA = .1216+001

L, R R = .6818+006

G, L R T = .9160+007

G, R R T = .1361+007

G, R R = .2799+006

L, R T = .3660+007

L, R R = .6529+006

N = .3550+001

T = .2639+007

G = .2201+001

BUMST PALSURL = .1459+005

STRAIN = .3947+000

14

POISSON, M = .3688+000  
G, F = .7917+007  
G, M = .1150+006  
L, L = .9657+007  
L, T = .6196+006  
POISSON, LT = .2898+000  
POISSON, TL = .1824+001  
POISSON, TT = .3890+000  
G, LT = .3476+006  
L, ME = .3192+006  
G, LL = .9828+007  
G, TT = .1388+006  
G, LT = .2933+006  
G, LR = .2927+006  
G, IR = .2230+006  
LRBDAE = .1192+001  
G, RR = .1388+006  
G, ITH = .4208+007  
G, IUP = .1367+007  
G, IHR = .2930+006  
L, IH = .3110+007  
L, IR = .7080+006  
N = .5834+001  
F = .2754+007  
G = .4320+007  
BULK PRESSURE = .1488+005  
STRAIN = .5259+001

C-2

POISSON, M: .3636+000

G, F: .7917+007

G, M: .1201+006

L, L: .9664+007

L, F: .6439+006

POISSON, L: .2818+000

POISSON, IL: .1877+001

POISSON, II: .3825+000

L, L: .3624+006

L, M: .5275+006

L, LL: .9832+007

L, LI: .7633+006

L, L: .2989+006

L, IN: .2976+006

G, IR: .2329+006

L, MR: .1192+001

L, RR: .7633+006

L, TH: .4229+007

L, THS: .1367+007

C, TH: .2983+006

L, IN: .3732+007

L, M: .2315+006

N: .3388+001

F: .2801+007

G: .2371+007

BURST PRESSURE: .1500+005

STRAIN: .7062+002



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POISSON, M = .1591+000  
G, F = .7917+007

G, M = .1244+006

I, F = .9669+007

I, F = .6647+006

POISSON, L F = .2796+000

POISSON, L F = .1922+001

POISSON, L F = .3770+000

G, L F = .3799+006

I, M = .3382+006

C, L L = .9839+007

C, L F = .7894+006

C, L F = .3035+006

C, L F = .3015+006

G, L F = .2913+006

LAMBDA = .1186+001

C, M R = .7642+006

C, M F = .4247+007

C, M F = .1368+007

C, M F = .3025+006

C, M F = .3751+007

C, M F = .7218+006

N = .3351+001

F = .2841+007

G = .2515+007

BURST PRESSURE = .1511+005

STRAIN = .9489+003

POISSON, M: .1582+000

G, F: .1917+007

G, M: .1291+006

L, L: .9675+007

L, T: .6876+006

POISSON, L: .2771+000

POISSON, IL: .1969+001

POISSON, IT: .3711+000

G, L: .3888+006

L, M: .1502+006

C, IL: .9896+007

C, IT: .8071+006

C, L: .3083+006

C, M: .3056+006

G, IR: .2508+006

LAMBDA: .1180+001

C, UR: .8071+006

C, IMIN: .9266+007

C, IHS: .1168+007

C, IHI: .3069+006

L, IN: .3772+007

L, M: .2737+006

N: .3112+001

P: .2083+007

G: .2861+007

BURST PRESSURE: .1572+005

STRAIN: .1275+003

P	POISSON	ELAS MOD	MURST P	PST	SFS VEC	SIN VEC
.670-002	.433903+000	.150502+006	.131062+005	.210411-001	.212385+004	.303109-002
.500-001	.380771+000	.286177+006	.145946+005	.197334-001	.715432+003	.923369-003
.308+000	.369777+000	.314910+006	.140010+005	.193535-001	.420959+003	.543404-003
.272+001	.363563+000	.327494+006	.150038+005	.191952-001	.306242+003	.385190-003
.201+002	.359112+000	.336242+006	.151076+005	.190633-001	.202421+003	.253279-003
.148+003	.35164+000	.350160+006	.152217+005	.189204-001	.802976+002	.110354-003

INPUT A MATRIX

.5000+000	.8008+000	.9820+000	.9775+000	.9997+000	.1000+001
.1192+000	.5000+000	.8808+000	.9820+000	.9975+000	.9997+000
.1799+001	.1192+000	.5000+000	.8808+000	.9975+000	.9997+000
.2473+002	.1799+001	.1192+000	.5000+000	.8808+000	.9820+000
.3323+003	.2473+002	.1799+001	.1192+000	.5000+000	.9820+000
.4540+004	.3323+003	.2473+002	.1799+001	.1192+000	.5000+000

INPUT F MATRIX

.2124+004	.7154+003	.4290+003	.5062+003	.2024+003	.0830+002
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INPUT S MATRIX

.3031+002	.0238+003	.5435+003	.3852+003	.2533+003	.1104+003
-----------	-----------	-----------	-----------	-----------	-----------

INPUTED MATRIX

.3753+001	.8680+001	.1003+002	.1019+002	.9851+001	.7506+001
.1175+001	.8469+001	.1181+002	.1317+002	.1291+002	.9851+001
.1837+000	.1599+001	.0952+001	.1275+002	.1317+002	.1019+002
.2526+001	.2412+000	.1651+001	.0952+001	.1181+002	.1003+002
.3308+002	.3199+001	.2412+000	.1599+001	.0952+001	.8680+001
.3408+003	.3303+002	.2526+001	.1651+001	.1175+001	.3753+001

SOLUTION G MATRIX

.4275+004	.2445+003	.3141+002	.1411+003	.1407+003
-----------	-----------	-----------	-----------	-----------

SUM OF G: .4189+004

A CONSTANT: .1117+005

SOLUTION H MATRIX

.6555+002	.1116+002	.3531+003	.3813+004	.1764+003	.1757+003
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SUM OF H: .6182+002

AA CONSTANT: .2499+001

CASE 2  
\*\*\*\*\*

V.F. = .1900+000

POISSON'S RATIO = .2000+000

HEAT = .1100+001

K IN LBS = .1920+001

V.F. = .6000+000

V.F. = .9000+000

INITIAL BURST PRESSURE = .1863+005

INITIAL STRAIN = .1546+001

POISSON, M = .4339+000

G, F = .7917+007

G, M = .5530+005

C, L = .1146+008

I, I = .3916+006

POISSON, L = .2936+000

POISSON, IL = .1003+001

POISSON, IT = .4251+000

G, L = .2453+006

I, M = .1506+008

C, LL = .1158+008

C, IT = .4819+006

C, LI = .2020+006

C, IR = .2067+006

G, IR = .1374+006

LABOR = .1233+001

C, MR = .4819+006

C, ITH = .4697+007

C, IHP = .1537+007

C, IHR = .2043+006

C, IM = .4154+007

I, M = .4960+008

M = .4469+001

F = .2319+007

G = .1983+007

BURST PRESSURE = .1600+005

STRAIN = .2687+001

POISSON, H = .1808+000

G, E = .7917+007

G, M = .1036+006

L, L = .1151+008

L, E = .6994+006

POISSON, L E = .7723+003

POISSON, L E = .1654+001

POISSON, L E = .3640+000

G, L E = .4456+006

L, M E = .2861+006

C, L L = .1168+008

C, L E = .0191+006

C, L E = .3038+006

C, L R = .3014+006

G, L R = .2564+006

LAMBDA = .1169+001

C, R R = .0191+006

C, L H M = .4984+007

C, L H S = .1567+007

C, L H M = .3026+006

L, L H = .4437+007

L, M E = .1862+006

N = .3564+001

F = .3099+007

G = .2703+007

HURST PRESSURE = .1774+005

STRAIN = .3246+000

POISSON, M: .3688+000

G, F: .1917+007

G, M: .1150+006

L, F: .1153+006

L, L: .7682+006

POISSON, L: .2675+000

POISSON, R: .1783+001

POISSON, R: .3517+000

G, L: .4913+006

L, M: .3192+006

L, L: .1173+008

L, L: .8855+006

L, L: .3213+006

C, R: .3172+006

G, R: .2893+006

LANBDA: .1158+001

C, R: .8855+006

C, M: .5045+007

C, M: .1562+007

C, M: .3198+006

L, L: .4501+007

L, R: .8596+006

M: .3442+001

L: .3244+007

S: .2852+007

PURSE PRESSURE: .1010+005

SINAI: .4325+001



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POISSON, M: +3636+000  
G, F: +7917+007  
G, M: +1201+006  
I, L: +1151+008  
I, F: +7981+006  
POISSON, L: +2659+000  
POISSON, IL: +1837+001  
POISSON, IF: +3965+000  
G, L: +5112+006  
L, M: +3275+006  
C, LL: +1171+008  
C, IF: +9163+006  
C, L: +3291+006  
C, IM: +3235+006  
G, IM: +2969+006  
L, MBUA: +1159+001  
C, ME: +9163+006  
C, IMIM: +5071+007  
C, IMPS: +1569+007  
C, IMH: +3263+006  
I, IM: +9529+007  
I, ME: +8892+006  
M: +1395+001  
F: +3305+007  
G: +2916+007  
BUNST PRESSON: +1825+005  
SMAIM: +5807+002

POISSON, M: .1591+000

G, F: .1917+000

G, M: .1798+006

F, L: .1158+008

L, F: .8216+006

POISSON, L, I: .2616+000

POISSON, IL: .1882+001

POISSON, I, I: .3922+000

G, L, I: .5283+006

G, M: .3382+006

G, L, I: .1171+008

G, I, I: .9829+006

G, L, I: .3351+006

G, I, I: .3288+006

G, I, I: .3088+006

LAMBDA: .1150+001

G, M: .9829+006

G, I, I, I, I: .5091+007

G, I, I, I, I, I: .1568+007

G, I, I, I, I: .3319+006

L, I, I, I: .9551+007

L, I, I: .9093+006

M: .3356+001

F: .3356+007

G: .2910+007

POISSON PRESSURE: .1037+0/5

SIMBIM: .1602+001

POISSON, ME .3542+000

G, F .7917+007

G, ME .3293+006

F, LE .8154+008

F, FZ .8519+006

POISSON, LI .2617+000

POISSON, IL .1932+001

POISSON, IT .3374+000

G, LI .5473+006

G, ME .3502+006

C, LL .1172+008

C, IT .9712+006

C, LI .3416+006

C, IR .3343+006

G, IR .3185+006

LAMBDAE .1126+001

C, ME .9712+006

C, INIR .5118+007

C, IMPSE .1562+007

C, INIR .3374+006

F, ME .4574+007

F, ME .2221+006

ME .3316+001

F .3411+007

GE .1030+007

BURST PRESSURE .1451+005

STRAIN .1048+003

P	POISSON	ELAS MOD	BURST P	POST	STS VEC	STS VEC
-670-002	.433903+000	.158582+006	.159988+005	.180014-001	.263122+004	.254137-002
-500-001	.380771+000	.286077+006	.177432+005	.162316-001	.886824+003	.771592-003
-360+000	.368777+000	.14910+006	.180960+005	.159151-001	.533999+003	.455118-003
-272+001	.363563+000	.327494+006	.1824+005	.157835-001	.381043+003	.323453-003
-201+002	.359112+000	.338242+006	.183746+005	.156738-001	.255417+003	.213925-003
-148+003	.354164+000	.150160+006	.185148+005	.155551-001	.115215+003	.951358-004

INPUT A MATRIX

.5010+000	.8808+000	.9820+000	.9975+000	.9997+000	.1000+001
.1192+000	.5003+000	.8808+000	.9820+000	.9975+000	.9997+001
.1192+000	.5003+000	.8808+000	.9820+000	.9975+000	.9997+001
.2873+002	.1199+001	.1192+000	.5003+000	.8808+000	.9820+000
.3354+003	.2873+002	.1199+001	.1192+000	.5003+000	.8808+000
.4340+004	.3354+003	.2873+002	.1199+001	.1192+000	.5003+000

INPUT F MATRIX

.2631+004	.8868+003	.5340+003	.1830+003	.2554+003	.1152+003
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INPUT S MATRIX

.2541+002	.7716+001	.4551+001	.3225+001	.2138+001	.9514+004
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INVERTED MATRIX

.3753+001	.8660+001	.1003+002	.1019+002	.9851+001	.7506+001
.1175+001	.6869+001	.1181+002	.1317+002	.1291+002	.9851+001
.1817+000	.1599+001	.6952+001	.1225+002	.1317+002	.1019+002
.2526+001	.2812+000	.1657+001	.6952+001	.1181+002	.1003+002
.3305+002	.3149+001	.2812+000	.1599+001	.6869+001	.8660+001
.3408+003	.3305+002	.2526+001	.1817+000	.1175+001	.3753+001

SOLUTION G MATRIX

.5282+004	.2787+003	.2771+003	.6371+002	.1490+003	.1913+003
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SUM OF CV: .5185+004

A. CONSTANT: .1385+005

SOLUTION H MATRIX

.5502+002	.9326+003	.2830+003	.8858+004	.1130+003	.1555+003
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SUM OF HV: .5187+002

B. CONSTANT: .2065+001

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102

CASE 3  
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L, F = .1900+000

POISSON, F = .2000+000

W, F = .1100+001

N, THE TA = .1920-001

V, F = .7000+000

V, F = .3000+000

INITIAL BUNST PRESSURE = .2295+005

INITIAL STRAIN = .1255-001

POISSON, M: .4339+000

G, E: .7917+007

G, M: .5530+005

L, L: .1335+000

L, T: .5185+006

POISSON, L: .2702+000

POISSON, IL: .1050+001

POISSON, IT: .3690+000

G, L: .8971+006

L, M: .1585+006

G, LL: .1397+008

G, IT: .6090+006

G, L: .2290+006

G, IR: .2252+006

G, IR: .1899+006

LABOR, L: .1160+001

G, IR: .6090+006

G, IIR: .5557+007

G, IIR: .1709+007

G, IIR: .2296+006

L, IR: .8998+007

L, M: .5901+006

R: .4364+001

F: .2703+007

G: .2988+007

BURST PRESSURE: .1982+005

STRAIN: .2213+001

POISSON, M = .1608+000

G, F = .7917+007

G, M = .1036+006

L, L = .1339+008

L, T = .9212+006

POISSON, L1 = .2542+000

POISSON, L2 = .1750+001

POISSON, L3 = .3233+000

G, L1 = .7798+006

L, M = .2861+006

G, L2 = .1356+008

G, L3 = .1038+007

G, L4 = .1397+006

G, L5 = .3917+006

G, L6 = .3981+006

LAMBDA = .1132+001

G, M2 = .1038+007

G, M3 = .5955+007

G, M4 = .1699+007

G, M5 = .3962+006

L, M2 = .5909+007

L, M3 = .1006+007

M = .3977+001

F = .3782+007

G = .3935+007

BURST PRESSURE = .2177+005

STRAIN = .2646+000



POISSON, M = .3688+000

G, F = .7917+007

G, ME = .1150+006

L, L = .1339+008

L, L = .1011+007

POISSON, LT = .2506+000

POISSON, IL = .1891-001

POISSON, IT = .3181+000

G, LT = .8522+006

L, M = .3132+006

C, LL = .1350+008

C, IT = .1132+007

C, LL = .3249+006

C, IT = .3625+006

G, IT = .3845+006

LAMUDA = .1125+001

C, ME = .1132+007

C, IT IN = .6037+007

C, ITPS = .1149+007

C, ITN = .3685+006

L, IT = .5499+007

L, ME = .1026+007

N = .3359+001

F = .3912+007

G = .2631+007

BURST PRESSURE = .2224+005

STRAIN = .3518-001

POISSON, M = .3636+000  
G, F = .7917+007  
G, M = .1201+006  
L, L = .1340+008  
L, T = .1049+007  
POISSON, L, T = .491+000  
POISSON, L, T = .1951+001  
POISSON, T, T = .3102+000  
G, L, T = .0035+006  
L, M = .3275+006  
L, L, L = .1359+008  
L, T, T = .1172+007  
L, L, T = .3093+006  
L, T, R = .3710+006  
G, T, R = .4005+006  
LAMBDA = .1122+001  
C, R, R = .1172+007  
L, H, H, T = .6073+007  
L, H, H, S = .1692+007  
L, H, H, R = .3777+006  
L, H, T = .5530+007  
L, M = .1135+007  
N = .3313+001  
F = .4052+007  
G = .3713+007  
DENSE PRESSURE = .2744+005  
SLOPE = .0721+002

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POISSON, M = .5591+000

G, F = .7917+007

G, M = .1244+006

L, L = .1340+008

L, T = .1043+007

POISSON, L1 = .2477+000

POISSON, L2 = .2001+001

POISSON, T1 = .3069+000

G, L1 = .9101+006

L, M = .3302+006

C, L1 = .1360+008

C, T1 = .1206+007

L, L1 = .1922+006

C, L2 = .3781+006

G, L2 = .4142+006

LAMBDA = .1120+001

C, M1 = .1206+007

C, M2 = .6104+007

C, M3 = .1620+007

C, M4 = .3853+006

L, M1 = .5571+007

L, M2 = .1168+007

M = .3275+001

F = .4119+007

G = .3789+007

BURST PRESSURE = .2261+005

STRAIN = .6339+003

14

POISSON, M: .3547+000  
G, I: .7917+007  
G, M: .1203+006  
L, I: .1341+008  
L, I: .1119+007  
POISSON, L: .2462+000  
POISSON, IL: .2056+001  
POISSON, IT: .1033+000  
G, I: .2395+006  
L, M: .3502+006  
G, LL: .1340+008  
G, IT: .1244+007  
G, I: .2015+006  
G, IM: .3057+006  
G, IT: .2203+006  
LANDUA: .1110+001  
G, RR: .1244+007  
G, IM: .6138+007  
G, IHS: .1687+007  
G, IM: .3035+006  
L, IM: .5607+007  
L, M: .1205+007  
N: .3236+001  
F: .4197+007  
G: .3867+007  
BURST PRESSURE: .2200+105  
STRAIN: .4512+004

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P	POISSON	FLAS MUD	MURST P	POST	SIS VIC	SIN VIC
.670-002	.433903+000	.156582+006	.194214+005	.148290-001	.352858+004	.227899-002
.500-001	.340771+000	.246077+006	.217715+005	.132203-001	.117855+004	.678331-003
.368+000	.360777+000	.314910+006	.222420+005	.129490-001	.707177+003	.397996-003
.212+001	.363563+000	.327494+006	.274438+005	.128321-001	.506245+003	.282077-003
.201+002	.359112+000	.338242+006	.226133+005	.127359-001	.336747+003	.185894-003
.144+003	.354164+000	.350160+006	.227990+005	.126321-001	.150962+003	.821113-004

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## INPUT A MATRIX

.5000+000	.8808+000	.9420+000	.9975+000	.9997+000	.1000+001
.1192+000	.5000+000	.8808+000	.9975+000	.9997+000	.9997+000
.1799+001	.1192+000	.5000+000	.8808+000	.9975+000	.9975+000
.2831+002	.1799+001	.1192+000	.5000+000	.8808+000	.9975+000
.3328+003	.2831+002	.1799+001	.1192+000	.5000+000	.8808+000
.4580+004	.3328+003	.2831+002	.1799+001	.1192+000	.5000+000

## INPUT F MATRIX

.3529+004	.1179+004	.7072+003	.5062+003	.3367+003	.1510+003
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## INPUT S MATRIX

.2279+002	.6783+003	.3980+003	.2021+003	.1850+003	.8211+004
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## INVERTED MATRIX

.3753+001	.0680+001	.1003+002	.1019+002	.0851+001	.7506+001
.1175+001	.6869+001	.1181+002	.1317+002	.1291+002	.9851+001
.1837+000	.1599+001	.6952+001	.1225+002	.1317+002	.1019+002
.2526+001	.2912+000	.1657+001	.6952+001	.1181+002	.1003+002
.3305+002	.3199+001	.2812+000	.1599+001	.0680+001	.7506+001
.3808+003	.3305+002	.2526+001	.1837+000	.1175+001	.3753+001

## SOLUTION G MATRIX

.7133+004	.1066+004	.3776+003	.7869+002	.2033+003	.2400+003
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SUM OF GV: .6975+004

A CONSTANT: .1598+005

## SOLUTION H MATRIX

.4998+002	.8657+003	.2583+003	.3505+004	.1707+003	.1330+003
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SUM OF HV: .8680+002

AA CONSTANT: .1123+001

IN

## APPENDIX C

FORTRAN PROGRAM THESIS.CCC

三



[illegible]

CASE 1  
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LF = .1900+000

POISSON, F = .2000+000

POISSON, M = .3500+000

G, F = .7917+007

G, M = .1333+006

L, L = .9600+007

L, L = .7065+006

POISSON, L = .2750+000

POISSON, IL = .2007+001

POISSON, IIL = .3861+000

G, L = .4003+006

VF = .5000+000

VM = .5000+000

L, M = .3599+006

C, IL = .9051+007

L, IL = .0256+006

C, IL = .3119+006

C, IR = .7085+006

G, IR = .2586+006

LANRUA = .1175+001

C, RR = .0250+006

C, IRI = .4282+007

C, IRI = .1368+007

C, IRI = .3102+006

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LJUE .1789.007

LHE .7916.006

NE .1202.001

EE .2912.007

GE .2502.007

INITIAL BURST PRESSURE .1531.005

INITIAL STRAIN .1001.001

CASE 2  
\*\*\*\*\*

LF .1900+000  
POISSON,FE .2000+000  
POISSON,ME .3500+000  
G,FE .7917+007  
G,ME .1153+006  
  
L,LE .1154+008  
L,IE .0749+006  
POISSON,LT .2600+000  
POISSON,IL .1970+001  
POISSON,ITE .1335+000  
G,LT .5629+006  
  
VF .6000+000  
VM .4000+000  
L,ME .3599+006  
C,LT .1172+008  
C,IF .9945+006  
C,IT .3466+006  
C,TR .3584+006  
G,IR .3281+006  
LAMUDA .1143+001  
C,IR .9945+006  
C,IRIN .5134+007  
C,IRIS .1568+007  
C,IRIN .3425+006

11

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117

1. IN: 4601-007

1. RE: 4596-006

2. IN: 4284-001

3. IN: 4554-007

4. IN: 4079-007

INITIAL BURST PRESSURE: 1063.005

INITIAL STRAIN: 1586-001

CASE 3  
\*\*\*\*\*

LF = .1900+000

POISSON, L = .2000+000

POISSON, M = .3500+000

G, F = .7917+007

G, M = .1333+006

L, L = .1341+000

L, L = .1349+007

POISSON, L = .2450+000

POISSON, L = .2099+001

POISSON, L = .3003+000

L, L = .9634+006

VF = .7000+000

VM = .3000+000

L, M = .3599+006

C, L = .1361+000

C, L = .1275+007

C, L = .4083+006

C, L = .3915+006

G, L = .4418+006

LAMBDA = .3116+001

C, M = .1275+007

C, M = .6165+007

C, M = .1645+007

C, M = .3939+006

11

L INE -5617007

L INE -1239+007

NE -1206+001

FE -9299+007

GE -3925+007

INITIAL BURST PRESSURE -2495+005

INITIAL STRAIN -1255+001

4P41.5 MIN

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